



Neutrino Phenomenology

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Part 3

Neutrino Masses and the $\bar{\nu} \stackrel{?}{=} \nu$ Question

Are Neutrinos Their
Own Antiparticles?

What Is the Question?

For each *mass eigenstate* ν_i , does —

- $\bar{\nu}_i = \nu_i$ (Majorana neutrinos)

or

- $\bar{\nu}_i \neq \nu_i$ (Dirac neutrinos) ?

Equivalently, is the **Lepton Number L** defined by—

$$L(\nu) = L(\ell^-) = -L(\bar{\nu}) = -L(\ell^+) = 1 \text{ conserved?}$$

L is a leptonic analogue of the **Baryon Number B** that distinguishes the \bar{n} from the **n**.

If **L** is not conserved, then nothing distinguishes $\bar{\nu}_i$ from ν_i . We then have Majorana neutrinos.

Neutrino Masses

For a *Dirac* neutrino mass eigenstate ν of mass m , the mass term in the Lagrangian density is —

$$L_m = -m\bar{\nu}\nu$$

Then —

$$\langle \nu \text{ at rest} | H_m | \nu \text{ at rest} \rangle = \langle \nu \text{ at rest} | m \int d^3x \bar{\nu}\nu | \nu \text{ at rest} \rangle = m$$

↑
Hamiltonian

For a *Majorana* neutrino mass eigenstate ν of mass m , the mass term in the Lagrangian density is —

$$L_m = -\frac{m}{2} \bar{\nu} \nu$$

with $\nu^c = \underbrace{(\text{phase factor}) \times \nu}_{\text{Antineutrino} = \text{Neutrino}}$ Charge conjugate of ν

Then —

$$\langle \nu \text{ at rest} | H_m | \nu \text{ at rest} \rangle = \langle \nu \text{ at rest} | \frac{m}{2} \int d^3x \bar{\nu} \nu | \nu \text{ at rest} \rangle = m$$

{The matrix element of $\bar{\nu} \nu$ is doubled in the *Majorana* case.}

Chiral fields:

Chirally left- and right-handed fermion fields satisfy the constraints —

$$P_L f_L \equiv \frac{(1 - \gamma_5)}{2} f_L = f_L \quad \text{and} \quad P_R f_R \equiv \frac{(1 + \gamma_5)}{2} f_R = f_R$$

For a *massless* fermion, chirality = helicity.

In the Standard Model (SM), only chirally left-handed fermion fields couple to the W boson.

Therefore, it is convenient to express the SM in terms of “*underlying*” chiral fields.

Expressed in terms of chiral fields, any mass term connects only fields of *opposite* chirality:

$$\bar{g}_R f_L$$


Chiral fermion fields

$$\bar{j}_L k_L = \bar{j}_R k_R = 0$$


Chiral fermion fields

For example —

$$\bar{j}_L k_L = \overline{\left(\frac{1-\gamma_5}{2}\right)j} \left(\frac{1-\gamma_5}{2}\right)k = \bar{j} \left(\frac{1+\gamma_5}{2}\right) \left(\frac{1-\gamma_5}{2}\right)k = 0$$

Note: Charge conjugating a chiral field reverses its chirality.

Dirac Mass Term

For quarks, charged leptons and *maybe* neutrinos.

Suppose ν_L^0 and ν_R^0 are underlying chiral fields in terms of which the SM, extended to include neutrino mass, is written.

The **Dirac** mass term is then —

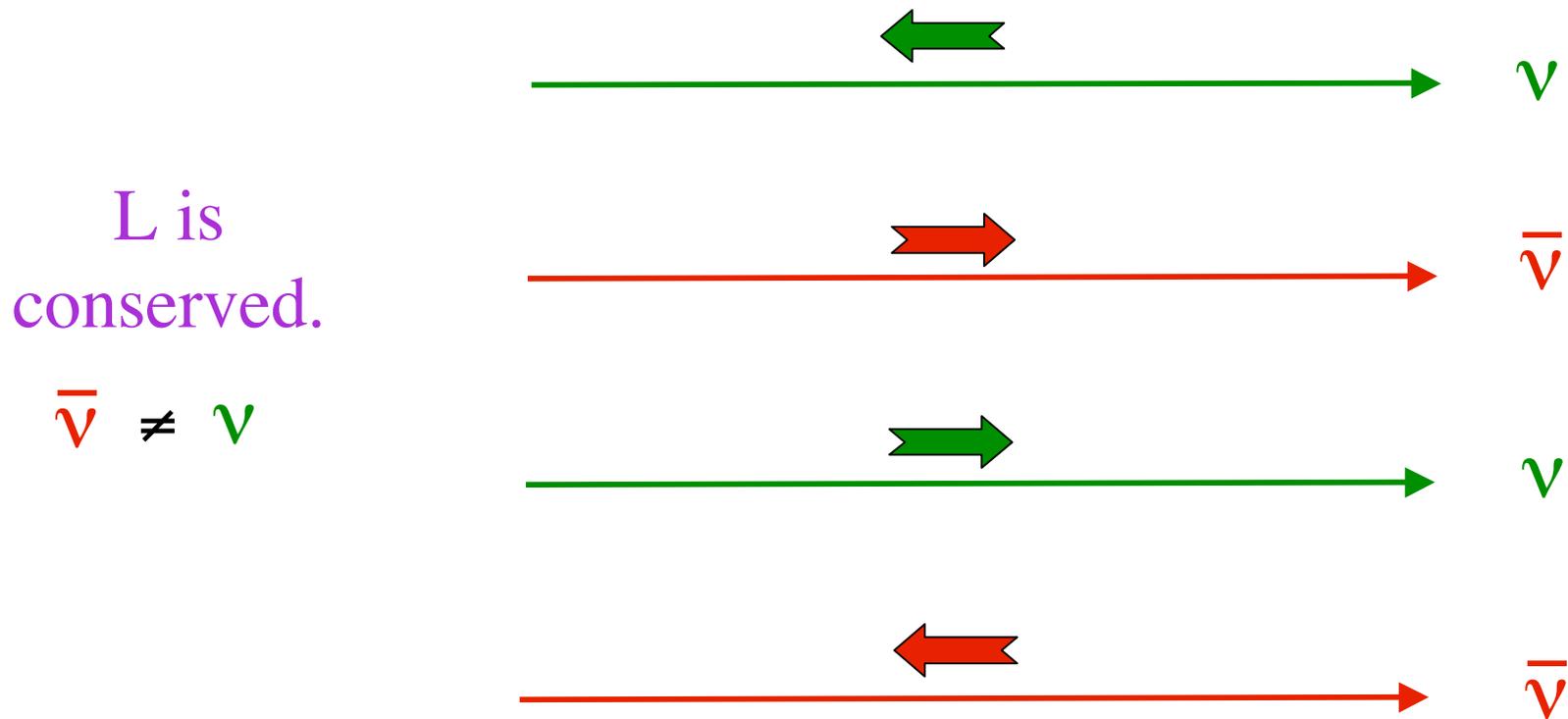
$$L_D = -m_D \overline{\nu_R^0} \nu_L^0 + \text{h.c.} = -m_D (\overline{\nu_R^0} \nu_L^0 + \overline{\nu_L^0} \nu_R^0)$$

In terms of $\nu \equiv \nu_L^0 + \nu_R^0$, $L_D = -m_D \bar{\nu} \nu$, since

$$\bar{\nu} \nu = \overline{(\nu_L^0 + \nu_R^0)} (\nu_L^0 + \nu_R^0) = \overline{\nu_R^0} \nu_L^0 + \overline{\nu_L^0} \nu_R^0$$

ν is the mass eigenstate, and has mass m_D .

We have 4 mass-degenerate states:



This collection of 4 states is a Dirac neutrino plus its antineutrino.

Majorana Mass Term

For neutrinos only.

Suppose ν_R^0 is a chirally right-handed field.

The **right-handed Majorana** mass term is then —

$$L_R = -\frac{m_R}{2} \overline{(\nu_R^0)^c} \nu_R^0 + \text{h.c.} = -\frac{m_R}{2} \left[\overline{(\nu_R^0)^c} \nu_R^0 + \overline{\nu_R^0} (\nu_R^0)^c \right]$$

In terms of $\nu \equiv \nu_R^0 + (\nu_R^0)^c$, $L_R = -\frac{m_R}{2} \bar{\nu} \nu$, since

$$\bar{\nu} \nu = \overline{\left[\nu_R^0 + (\nu_R^0)^c \right]} \left[\nu_R^0 + (\nu_R^0)^c \right] = \overline{(\nu_R^0)^c} \nu_R^0 + \overline{\nu_R^0} (\nu_R^0)^c$$

ν is the mass eigenstate, and has mass m_R .

$$\nu^c = \left[\nu_R^0 + (\nu_R^0)^c \right]^c = (\nu_R^0)^c + \nu_R^0 = \nu$$

Thus, ν is its own antiparticle. It is a Majorana neutrino.

We have only 2 mass-degenerate states:



L is *not* conserved:

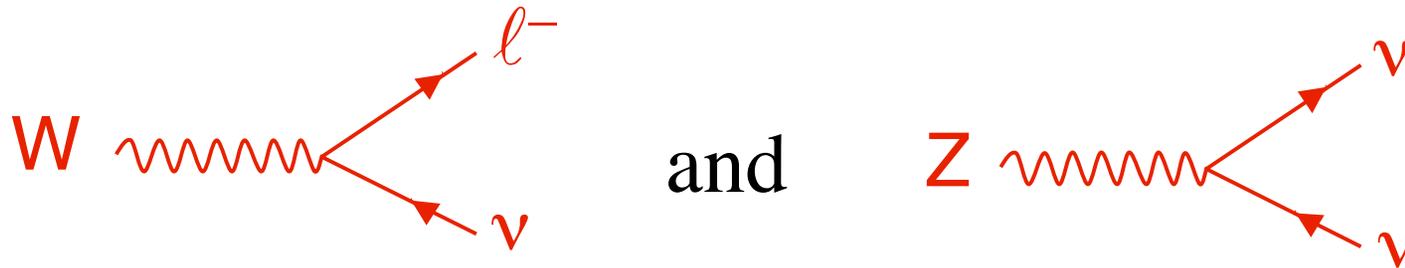


*Majorana
mass*

Why Many Theorists
Expect Majorana Masses

Hence, L – Nonconservation
and Majorana Neutrinos

The S(andard) M(odel)

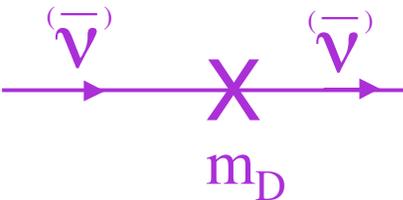


couplings conserve the **Lepton Number L**.

So do the Dirac charged-lepton mass terms

$$m_\ell \bar{l}_R l_L \quad \xrightarrow{l^{(\mp)}} \quad \text{X} \quad \xrightarrow{l^{(\mp)}} \quad m_\ell$$

- Original SM: $m_\nu = 0$.
- Why not add a **Dirac** mass term,

$$m_D \bar{\nu}_R \nu_L$$


Then everything conserves L , so for each mass eigenstate ν_i ,

$$\bar{\nu}_i \neq \nu_i \quad (\text{Dirac neutrinos})$$

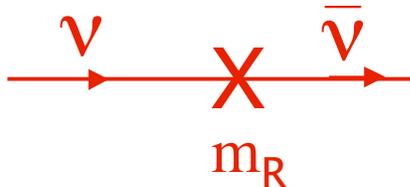
$$[L(\bar{\nu}_i) = -L(\nu_i)]$$

- The SM contains no ν_R field, only ν_L .

To add the Dirac mass term, we had to add ν_R to the SM.

Unlike ν_L , ν_R carries no Electroweak Isospin.

Thus, no SM principle prevents the occurrence of the **Majorana** mass term

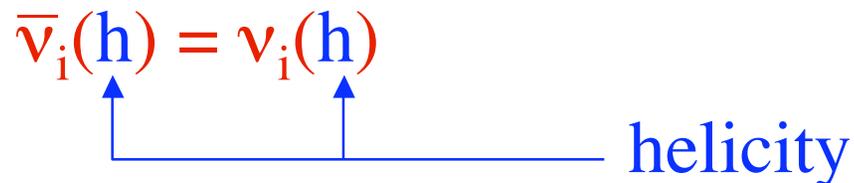
$$m_R \overline{\nu_R^c} \nu_R$$


This does not conserve L, so now

$$\bar{\nu}_i = \nu_i \quad (\text{Majorana neutrinos})$$

[No conserved L to distinguish $\bar{\nu}_i$ from ν_i]

We note that $\bar{\nu}_i = \nu_i$ means —

$$\bar{\nu}_i(\mathbf{h}) = \nu_i(\mathbf{h})$$


This Leads Many Theorists To Expect Majorana Masses

The Standard Model (SM) is defined by the fields it contains, its **symmetries** (notably Electroweak Isospin Invariance), and its renormalizability.

Leaving neutrino masses aside, anything allowed by the SM symmetries occurs in nature.

If this is also true for neutrino masses,
then neutrinos have *Majorana masses*.

- The presence of Majorana masses
- $\bar{\nu}_i = \nu_i$ (Majorana neutrinos)
- L not conserved

— are all equivalent

Any one implies the other two.

(Recent work: Hirsch, Kovalenko, Schmidt)

To Determine If
Neutrinos Are Their
Own Antiparticles

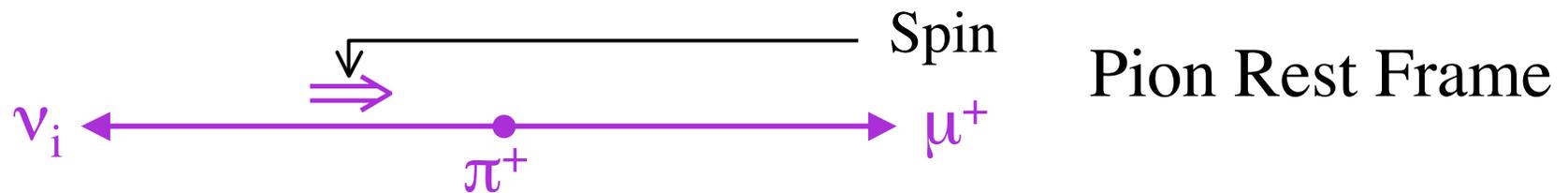
How Can We Demonstrate That $\bar{\nu}_i = \nu_i$?

We assume neutrino **interactions** are correctly described by the SM. Then the **interactions** conserve L ($\nu \rightarrow l^-$; $\bar{\nu} \rightarrow l^+$).

An Idea that Does Not Work

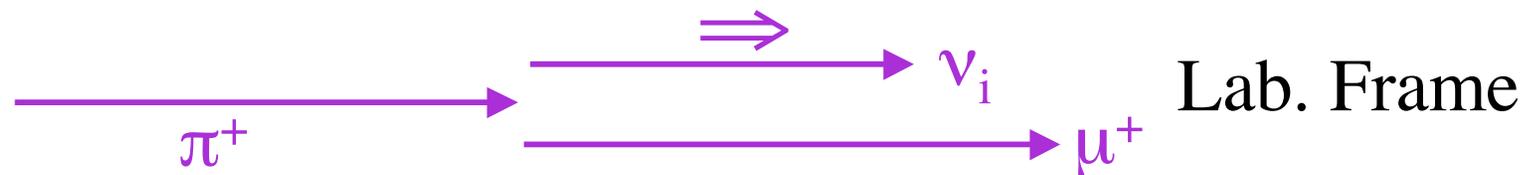
[and illustrates why most ideas do not work]

Produce a ν_i via—

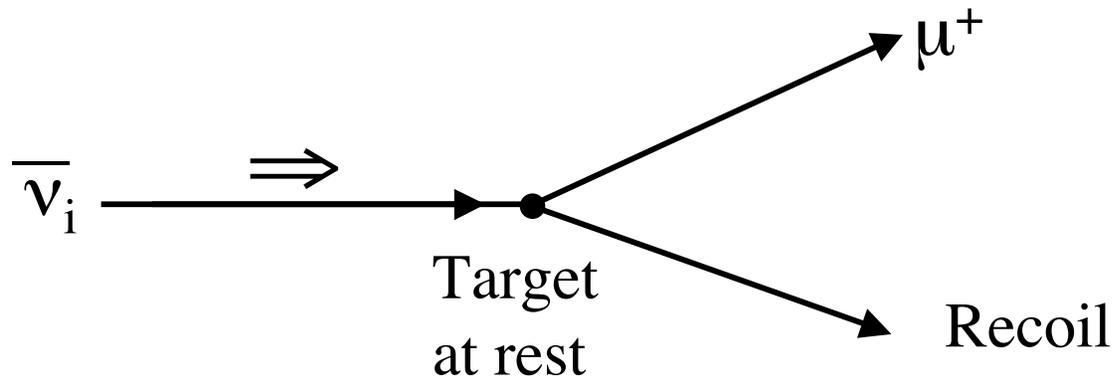


Give the neutrino a Boost:

$$\beta_\pi(\text{Lab}) > \beta_\nu(\pi \text{ Rest Frame})$$



The SM weak interaction causes—



$v_i = \bar{v}_i$ means that $v_i(\mathbf{h}) = \bar{v}_i(\mathbf{h})$.

↑ helicity

If $v_i \Rightarrow = \bar{v}_i \Rightarrow$,

our $v_i \Rightarrow$ will make μ^+ too.

Minor Technical Difficulties

$$\begin{aligned}\beta_{\pi}(\text{Lab}) &> \beta_{\nu}(\pi \text{ Rest Frame}) \\ \Rightarrow \frac{E_{\pi}(\text{Lab})}{m_{\pi}} &> \frac{E_{\nu}(\pi \text{ Rest Frame})}{m_{\nu_i}} \\ \Rightarrow E_{\pi}(\text{Lab}) &\gtrsim 10^5 \text{ TeV if } m_{\nu_i} \sim 0.05 \text{ eV}\end{aligned}$$

Fraction of all π – decay ν_i that get helicity flipped

$$\approx \left(\frac{m_{\nu_i}}{E_{\nu}(\pi \text{ Rest Frame})} \right)^2 \sim 10^{-18} \text{ if } m_{\nu_i} \sim 0.05 \text{ eV}$$

Since L-violation comes only from Majorana neutrino masses, any attempt to observe it will be at the mercy of the neutrino masses.

(BK & Stodolsky)

The Promising Approach — Neutrinoless Double Beta Decay [$0\nu\beta\beta$]

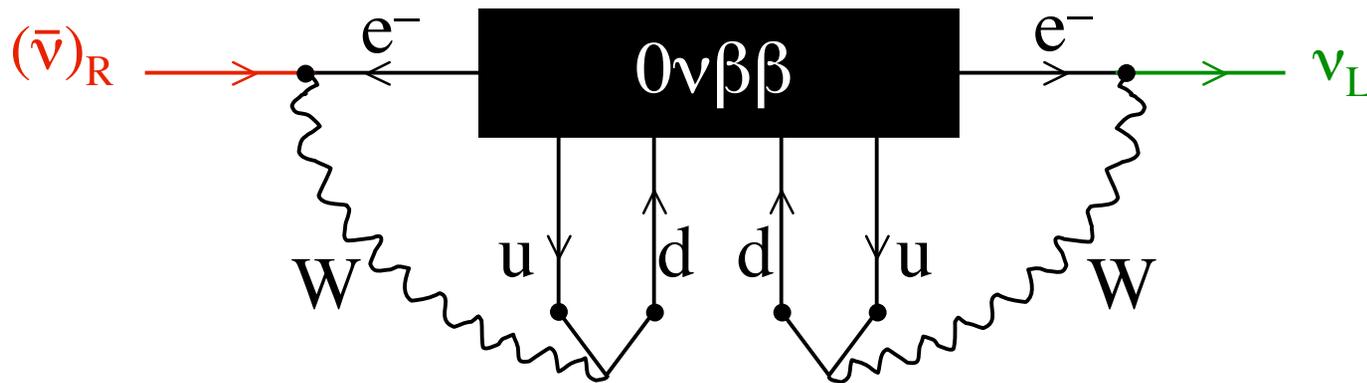


If we start with *a lot* of parent nuclei (say, one ton of them), we can cope with the smallness of \mathcal{L} .

Observation would imply \cancel{X} and therefore $\bar{\nu}_i = \nu_i$.

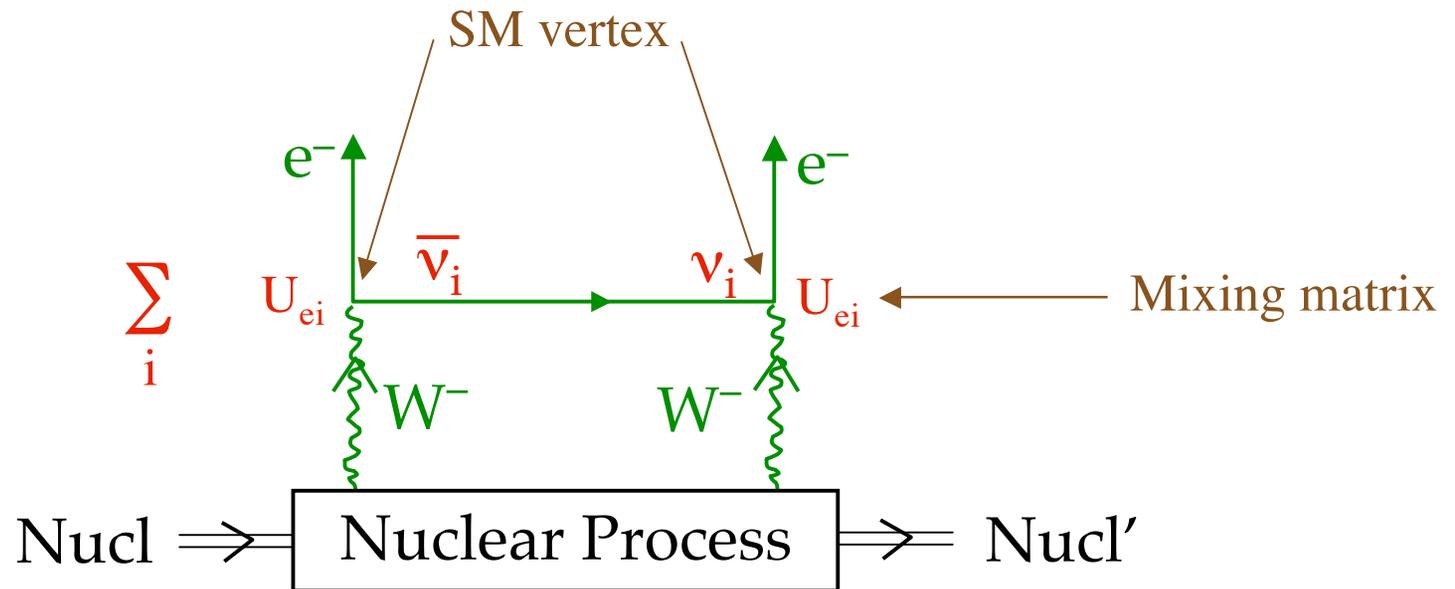
Whatever diagrams cause $0\nu\beta\beta$, its observation would imply the existence of a Majorana mass term:

Schechter and Valle

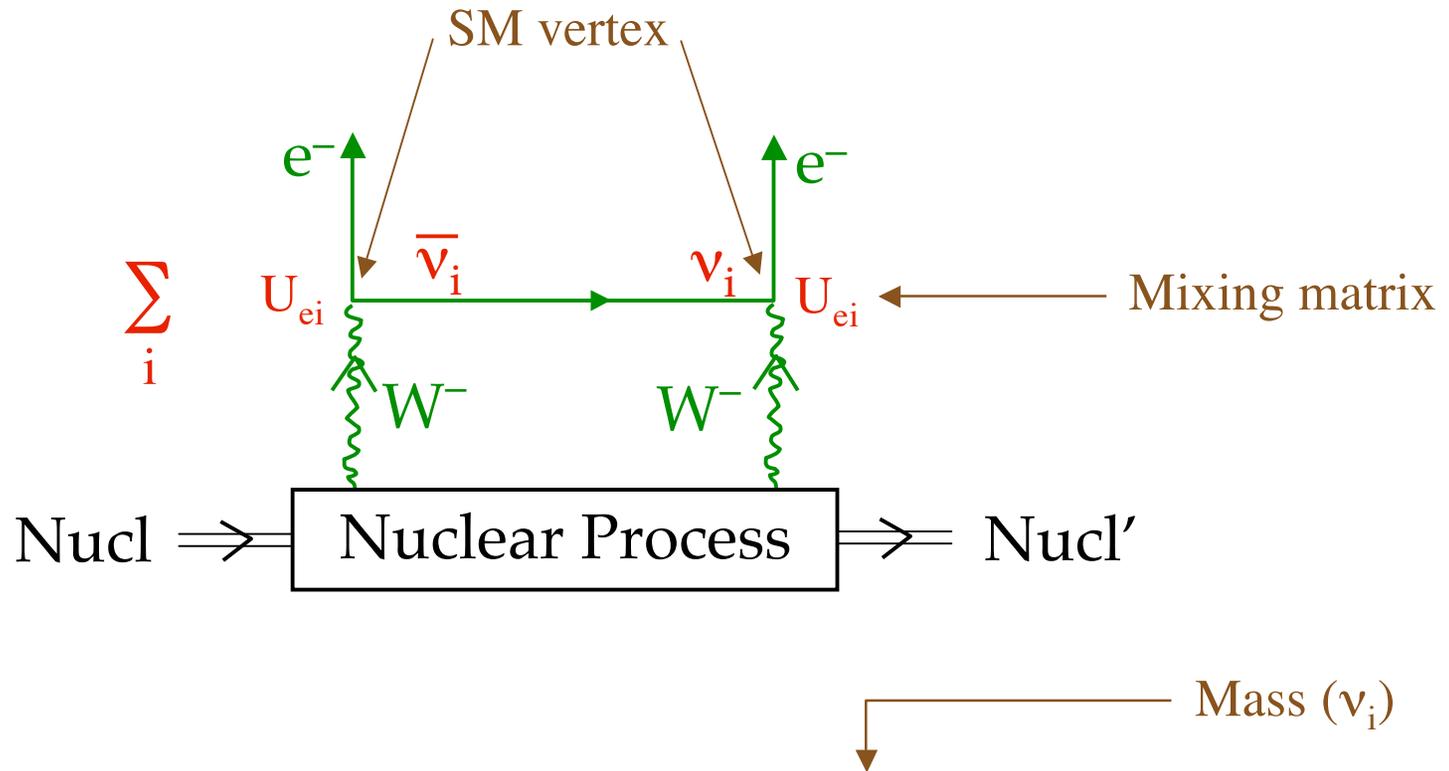


$(\bar{\nu})_R \rightarrow \nu_L$: A Majorana mass term

We anticipate that $0\nu\beta\beta$ is dominated by a diagram with Standard Model vertices:



In —



the $\bar{\nu}_i$ is emitted [RH + O{ m_i/E }LH].

Thus, Amp [ν_i contribution] $\propto m_i$

$$\text{Amp}[0\nu\beta\beta] \propto \left| \sum_i m_i U_{ei}^2 \right| \equiv m_{\beta\beta}$$

The proportionality of $0\nu\beta\beta$ to ν mass is no surprise.

$0\nu\beta\beta$ violates L. But the SM interactions conserve L.

The L – violation in $0\nu\beta\beta$ comes from underlying
Majorana neutrino mass terms.

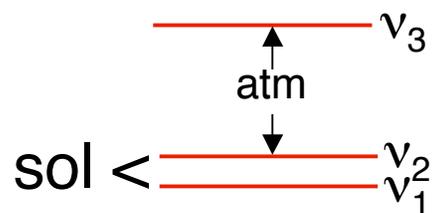
The $0\nu\beta\beta$ amplitude would be proportional to neutrino mass even if there were no helicity mismatch.

How Large is $m_{\beta\beta}$?

How sensitive need an experiment be?

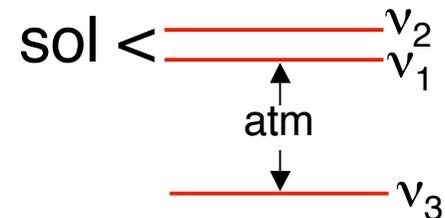
Suppose there are only 3 neutrino mass eigenstates. (More might help.)

Then the spectrum looks like —



Normal hierarchy

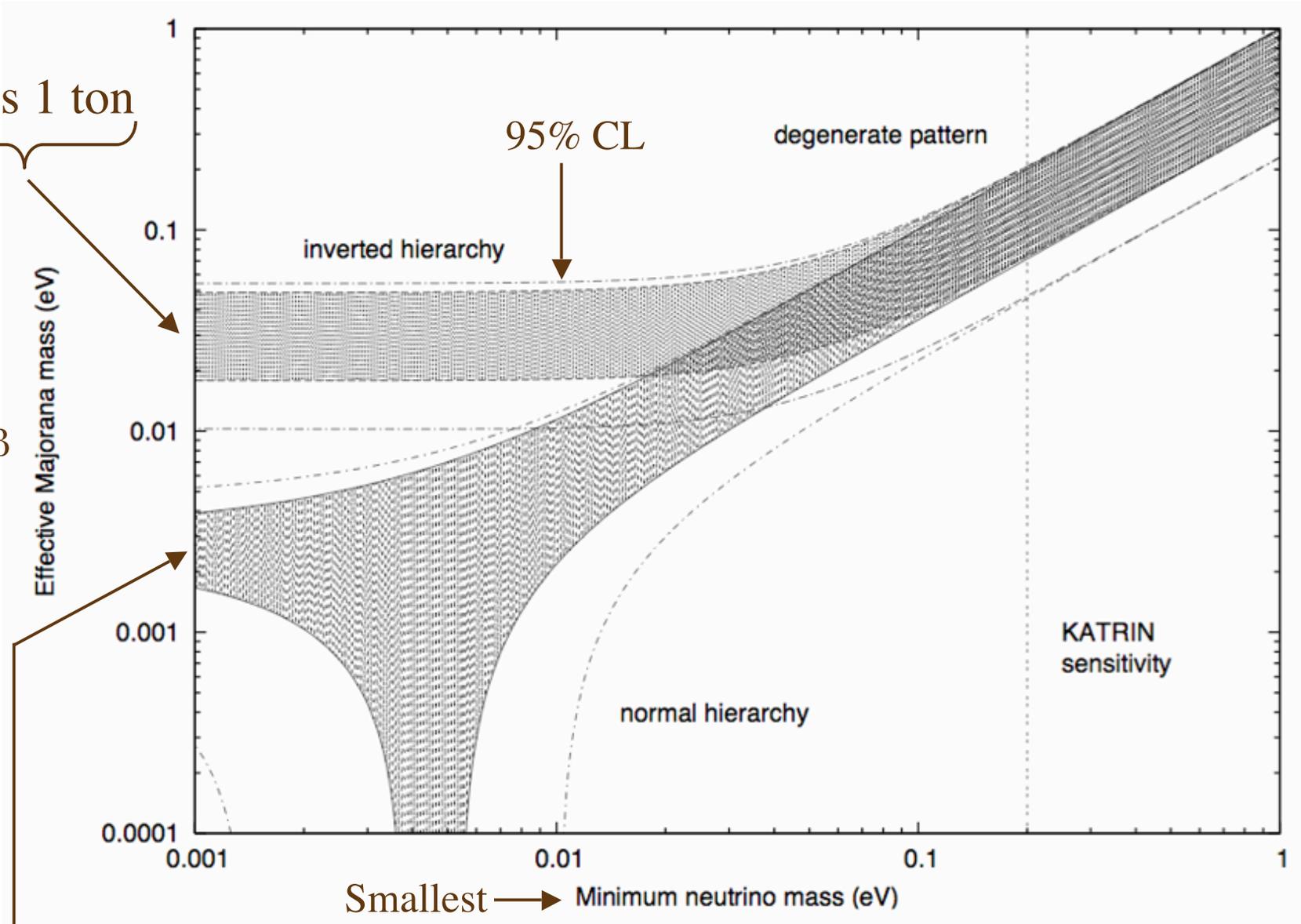
or



Inverted hierarchy

Takes 1 ton

$m_{\beta\beta}$

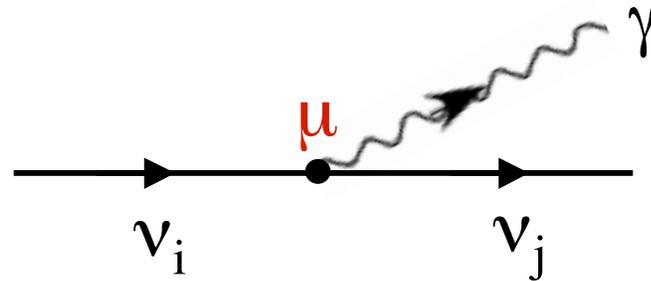


Takes
100 tons

$m_{\beta\beta}$ For Each Hierarchy

Possible Information From Neutrino Magnetic Moments

Both Majorana and Dirac neutrinos can have *transition* magnetic dipole moments μ :



For *Dirac* neutrinos, $\mu < 10^{-15} \mu_{\text{Bohr}}$

For *Majorana* neutrinos, $\mu < \text{Present bound}$

Present bound = $\begin{cases} 7 \times 10^{-11} \mu_{\text{Bohr}} ; \text{Wong et al. (Reactor)} \\ 3 \times 10^{-12} \mu_{\text{Bohr}} ; \text{Raffelt (Stellar E loss)} \end{cases}$

*An observed μ below the present bound but well above $10^{-15} \mu_{Bohr}$ would imply that neutrinos are *Majorana* particles.*

However, a dipole moment that large requires L-violating new physics below 100 TeV.

(Bell, Cirigliano, Davidson, Gorbahn, Gorchtein, Ramsey-Musolf, Santamaria, Vogel, Wise, Wang)

Neutrinoless double beta decay at the planned level of sensitivity only requires this new physics at $\sim 10^{15}$ GeV, near the Grand Unification scale.