- Electromagnetism is mediated by a massless photon, coupled to the electric charge;
- Mediator of charged-current weak interaction acquires a mass $M_W^2 = \pi\alpha / G_F \sqrt{2} \sin^2 \theta_W$,
- Mediator of (new!) neutral-current weak interaction acquires mass $M_Z^2 = M_W^2 / \cos^2 \theta_W$;
- Massive neutral scalar particle, the Higgs boson, appears, but its mass is not predicted;
- Fermions can acquire mass—values not predicted.

Expand about the vacuum state

Let $\phi = \begin{pmatrix} 0 \\ (v+\eta)/\sqrt{2} \end{pmatrix}$; in *unitary gauge*

$$\begin{aligned} \mathcal{L}_{\text{scalar}} &= \tfrac{1}{2}(\partial^\mu \eta)(\partial_\mu \eta) - \mu^2 \eta^2 \\ &\quad + \frac{v^2}{8}[g^2 \left|b^1_\mu - i b^2_\mu\right|^2 + (g' \mathcal{A}_\mu - g b^3_\mu)^2] \\ &\quad + \text{interaction terms} \end{aligned}$$

"Higgs boson" η has acquired (mass)^2 $M_H^2 = -2\mu^2 > 0$

$$\text{Define } W^\pm_\mu = \frac{b^1_\mu \mp i b^2_\mu}{\sqrt{2}}$$

$$\frac{g^2 v^2}{8}(\left|W^+_\mu\right|^2 + \left|W^-_\mu\right|^2) \Longleftrightarrow M_{W^\pm} = gv/2$$

$$(v^2/8)(g'\mathcal{A}_\mu - g b^3_\mu)^2 \ldots$$

Now define orthogonal combinations

$$Z_\mu = \frac{-g'\mathcal{A}_\mu + g b^3_\mu}{\sqrt{g^2 + g'^2}} \qquad A_\mu = \frac{g\mathcal{A}_\mu + g' b^3_\mu}{\sqrt{g^2 + g'^2}}$$

$$M_{Z^0} = \sqrt{g^2 + g'^2}\, v/2 = M_W \sqrt{1 + g'^2/g^2}$$

$\boxed{A_\mu \text{ remains massless}}$

$$\mathcal{L}_{\text{Yukawa}} = -\zeta_e \frac{(v+\eta)}{\sqrt{2}} (\bar{e}_R e_L + \bar{e}_L e_R)$$
$$= -\frac{\zeta_e v}{\sqrt{2}} \bar{e} e - \frac{\zeta_e \eta}{\sqrt{2}} \bar{e} e$$

electron acquires $m_e = \zeta_e v / \sqrt{2}$

Higgs-boson coupling to electrons: m_e / v ($\propto$ mass)

Desired particle content . . . plus a Higgs scalar

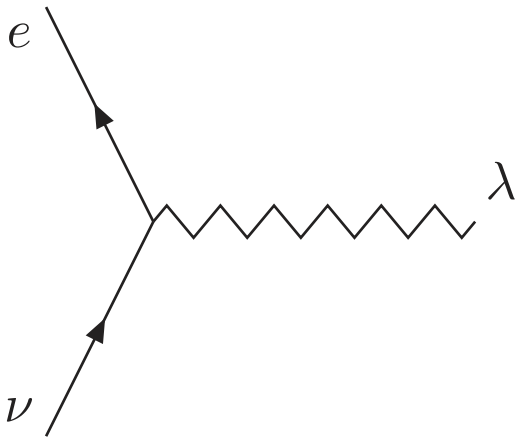
Values of couplings, electroweak scale v?

What about interactions?

Interactions . . .

$$\mathcal{L}_{W\text{-}\ell} = -\frac{g}{2\sqrt{2}}[\bar{\nu}\gamma^\mu(1-\gamma_5)eW_\mu^+ + \bar{e}\gamma^\mu(1-\gamma_5)\nu W_\mu^-]$$

$+$ similar terms for μ and τ



$$\frac{-ig}{2\sqrt{2}}\gamma_\lambda(1-\gamma_5)$$

W

$$= \frac{-i(g_{\mu\nu} - k_\mu k_\nu / M_W^2)}{k^2 - M_W^2}\,.$$

Compute $\nu_\mu e \to \mu\nu_e$

$$\sigma(\nu_\mu e \to \mu\nu_e) = \frac{g^4 m_e E_\nu}{16\pi M_W^4} \frac{[1-(m_\mu^2 - m_e^2)/2m_e E_\nu]^2}{(1+2m_e E_\nu/M_W^2)}$$

Reproduces 4-fermion result at low energies if

$$\frac{g^4}{16M_W^4} = 2G_F^2 \Rightarrow \frac{g}{2\sqrt{2}} = \left(\frac{G_F M_W^2}{\sqrt{2}}\right)^{\frac{1}{2}}$$

Using $M_W = gv/2$, determine the electroweak scale

$$\boxed{v = (G_F\sqrt{2})^{-\frac{1}{2}} \approx 246 \text{ GeV}}$$

$$\Rightarrow \langle\phi^0\rangle_0 = (G_F\sqrt{8})^{-\frac{1}{2}} \approx 174 \text{ GeV}$$

W-propagator modifies HE behavior

$$\sigma(\nu_\mu e \to \mu\nu_e) = \frac{g^4 m_e E_\nu}{16\pi M_W^4} \frac{[1-(m_\mu^2 - m_e^2)/2m_e E_\nu]^2}{(1+2m_e E_\nu/M_W^2)}$$

$$\lim_{E_\nu \to \infty} \sigma(\nu_\mu e \to \mu\nu_e) = \frac{g^4}{32\pi M_W^2} = \frac{G_F^2 M_W^2}{\sqrt{2}}$$

independent of energy!

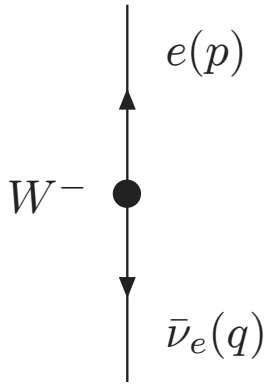
Partial-wave unitarity respected for

$$s < M_W^2[\exp(\pi\sqrt{2}/G_F M_W^2) - 1]$$

W-boson properties

No prediction yet for M_W (haven't determined g)

Leptonic decay $W^- \to e^- \nu_e$



$$p \approx \left(\frac{M_W}{2}; \frac{M_W \sin\theta}{2}, 0, \frac{M_W \cos\theta}{2}\right)$$

$$q \approx \left(\frac{M_W}{2}; -\frac{M_W \sin\theta}{2}, 0, -\frac{M_W \cos\theta}{2}\right)$$

$$\mathcal{M} = -i \left(\frac{G_F M_W^2}{\sqrt{2}}\right)^{\frac{1}{2}} \bar{u}(e,p)\gamma_\mu(1-\gamma_5)v(\nu,q)\,\varepsilon^\mu$$

$\varepsilon^\mu = (0;\hat{\varepsilon})$: W polarization vector in its rest frame

$$|\mathcal{M}|^2 = \frac{G_F M_W^2}{\sqrt{2}} \mathrm{tr}\left[\not{\varepsilon}(1-\gamma_5)\not{q}(1+\gamma_5)\not{\varepsilon}^* \not{p}\right] ;$$

$$\mathrm{tr}[\cdots] = \left[\varepsilon\cdot q\,\varepsilon^*\cdot p - \varepsilon\cdot\varepsilon^*\, q\cdot p + \varepsilon\cdot p\,\varepsilon^*\cdot q + i\epsilon_{\mu\nu\rho\sigma}\varepsilon^\mu q^\nu \varepsilon^{*\rho} p^\sigma\right]$$

$$\mathrm{tr}[\cdots] \;=\; [\varepsilon\cdot q\,\varepsilon^*\cdot p - \varepsilon\cdot\varepsilon^*\,q\cdot p + \varepsilon\cdot p\,\varepsilon^*\cdot q + i\epsilon_{\mu\nu\rho\sigma}\varepsilon^\mu q^\nu \varepsilon^{*\rho} p^\sigma]$$

decay rate is independent of W polarization; look first at longitudinal pol. $\varepsilon^\mu = (0;0,0,1) = \varepsilon^{*\mu}$, eliminate $\epsilon_{\mu\nu\rho\sigma}$

$$|\mathcal{M}|^2 = \frac{4G_F M_W^4}{\sqrt{2}}\sin^2\theta$$

$$\frac{d\Gamma_0}{d\Omega} = \frac{|\mathcal{M}|^2}{64\pi^2}\,\frac{\mathcal{S}_{12}}{M_W^3}$$

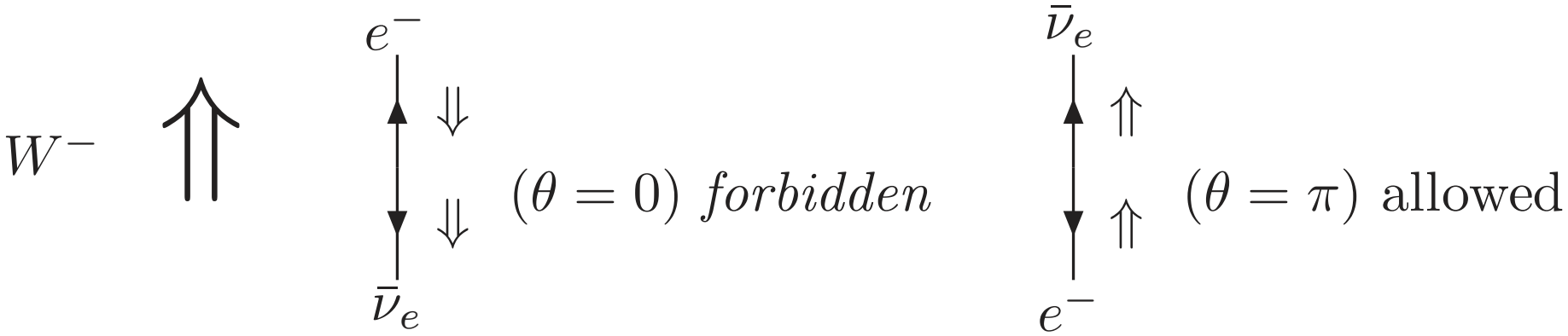
$$\mathcal{S}_{12} = \sqrt{[M_W^2 - (m_e+m_\nu)^2][M_W^2 - (m_e-m_\nu)^2]} = M_W^2$$

$$\frac{d\Gamma_0}{d\Omega} = \frac{G_F M_W^3}{16\pi^2\sqrt{2}}\sin^2\theta \qquad \boxed{\Gamma(W\to e\nu) = \frac{G_F M_W^3}{6\pi\sqrt{2}}}$$

Other helicities: $\varepsilon^{\mu}_{\pm 1} = (0; -1, \mp i, 0)/\sqrt{2}$

$$\frac{d\Gamma_{\pm 1}}{d\Omega} = \frac{G_F M_W^3}{32\pi^2\sqrt{2}}(1 \mp \cos\theta)^2$$

Extinctions at $\cos\theta = \pm 1$ are consequences of angular momentum conservation:



(situation reversed for $W^+ \to e^+\nu_e$)

e^+ follows polarization direction of W^+

e^- avoids polarization direction of W^-

important for discovery of $W^\pm$ in $\bar{p}p$ ($\bar{q}q$) $\mathcal{C}$ violation

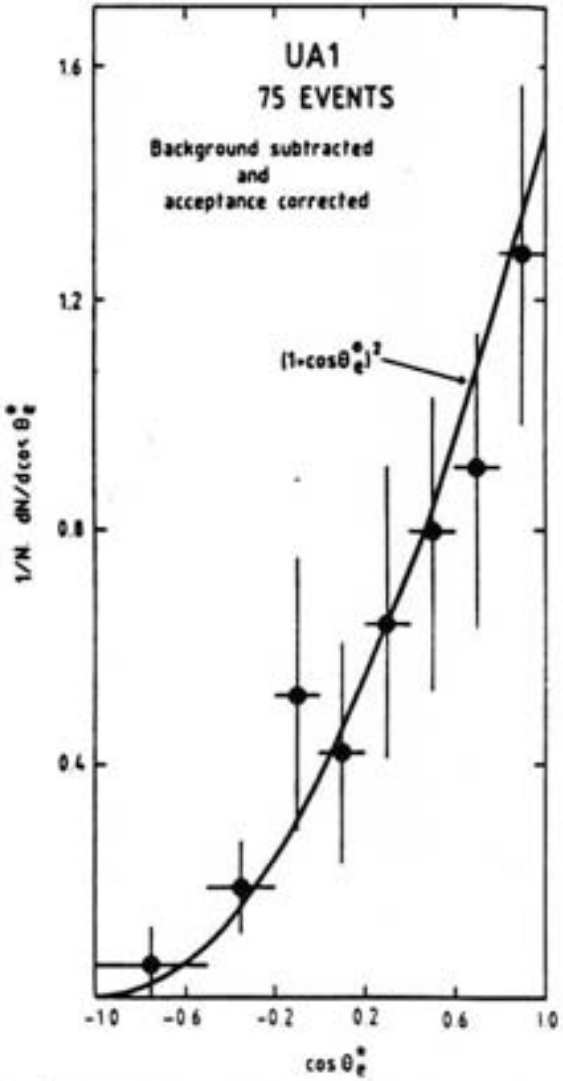


Fig. 2. The W decay angular distribution of the emission angle θ^* of the electron (positron) with respect to the proton (antiproton) direction in the rest frame of the W. Only those events for which the lepton charge and the decay kinematics are well determined have been used. The curve shows the $(V-A)$ expectation of $(1+\cos\theta^*)^2$.

Interactions . . .

$$\mathcal{L}_{A\text{-}\ell} = \frac{gg'}{\sqrt{g^2 + g'^2}} \bar{e}\gamma^\mu e A_\mu$$

. . . vector interaction; $\Rightarrow A_\mu$ as γ, provided we identify

$$\boxed{gg'/\sqrt{g^2 + g'^2} \equiv e}$$

Define $g' = g \tan\theta_W$ $\qquad$ θ_W: weak mixing angle

$$\begin{aligned} g &= e/\sin\theta_W \geq e \\ g' &= e/\cos\theta_W \geq e \end{aligned}$$

$$Z_\mu = b^3_\mu \cos\theta_W - \mathcal{A}_\mu \sin\theta_W \qquad A_\mu = \mathcal{A}_\mu \cos\theta_W + b^3_\mu \sin\theta_W$$

$$\mathcal{L}_{Z-\nu} = \frac{-g}{4\cos\theta_W} \bar{\nu}\gamma^\mu (1 - \gamma_5)\nu \; Z_\mu$$

Purely left-handed!

Interactions . . .

$$\mathcal{L}_{Z-e} = \frac{-g}{4\cos\theta_W}\bar{e}\left[L_e\gamma^\mu(1-\gamma_5) + R_e\gamma^\mu(1+\gamma_5)\right] e\, Z_\mu$$

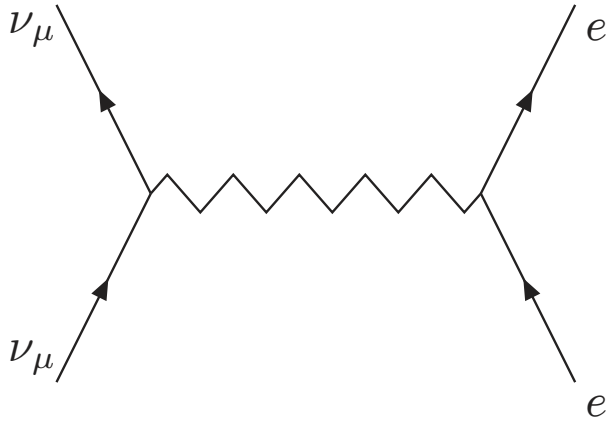
$$\begin{aligned} L_e &= 2\sin^2\theta_W - 1 = 2x_W + \tau_3 \\ R_e &= 2\sin^2\theta_W = 2x_W \end{aligned}$$

Z-decay calculation analogous to $W^\pm$

$$\begin{aligned} \Gamma(Z \to \nu\bar{\nu}) &= \frac{G_F M_Z^3}{12\pi\sqrt{2}} \\ \Gamma(Z \to e^+e^-) &= \Gamma(Z \to \nu\bar{\nu})\left[L_e^2 + R_e^2\right] \end{aligned}$$

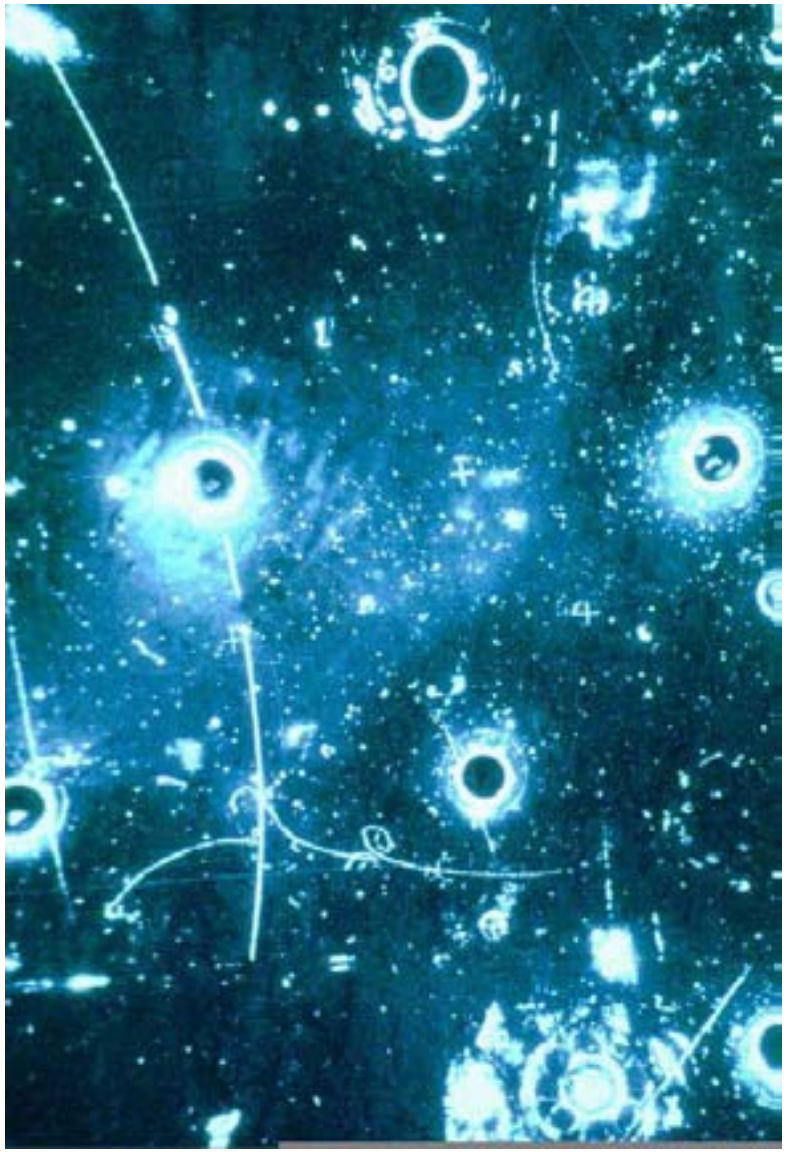
Neutral-current interactions

New νe reaction, not present in $V - A$



$$\begin{aligned}
\sigma(\nu_\mu e \to \nu_\mu e) &= \frac{G_F^2 m_e E_\nu}{2\pi}\left[L_e^2 + R_e^2/3\right] \\
\sigma(\bar{\nu}_\mu e \to \bar{\nu}_\mu e) &= \frac{G_F^2 m_e E_\nu}{2\pi}\left[L_e^2/3 + R_e^2\right] \\
\sigma(\nu_e e \to \nu_e e) &= \frac{G_F^2 m_e E_\nu}{2\pi}\left[(L_e+2)^2 + R_e^2/3\right] \\
\sigma(\bar{\nu}_e e \to \bar{\nu}_e e) &= \frac{G_F^2 m_e E_\nu}{2\pi}\left[(L_e+2)^2/3 + R_e^2\right]
\end{aligned}$$

Gargamelle $\bar{\nu}_\mu e$ event (1973)



- Electromagnetism is mediated by a massless photon, coupled to the electric charge;
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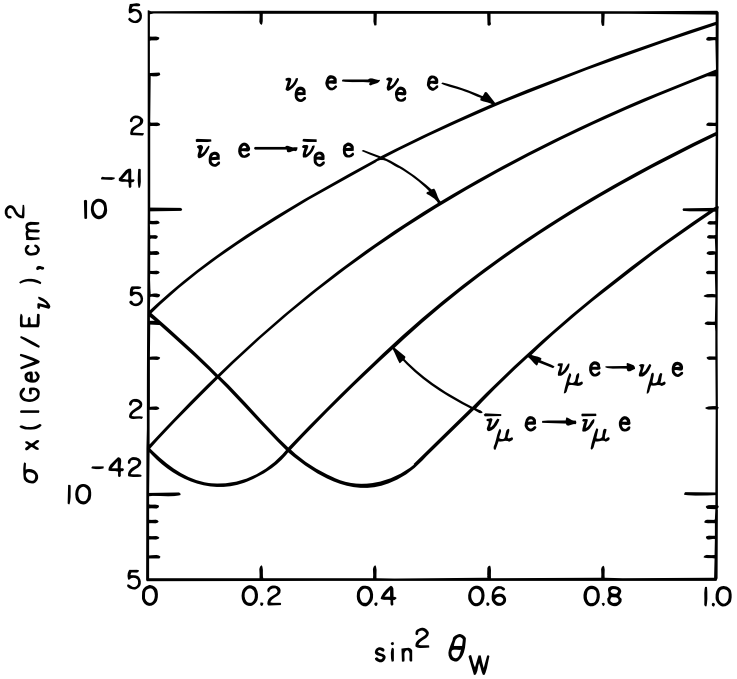
- Massive neutral scalar particle, the Higgs boson, appears, but its mass is not predicted;
- Fermions can acquire mass—values not predicted.

Determine $\sin^2\theta_W$ to predict M_W, M_Z

"Model-independent" analysis



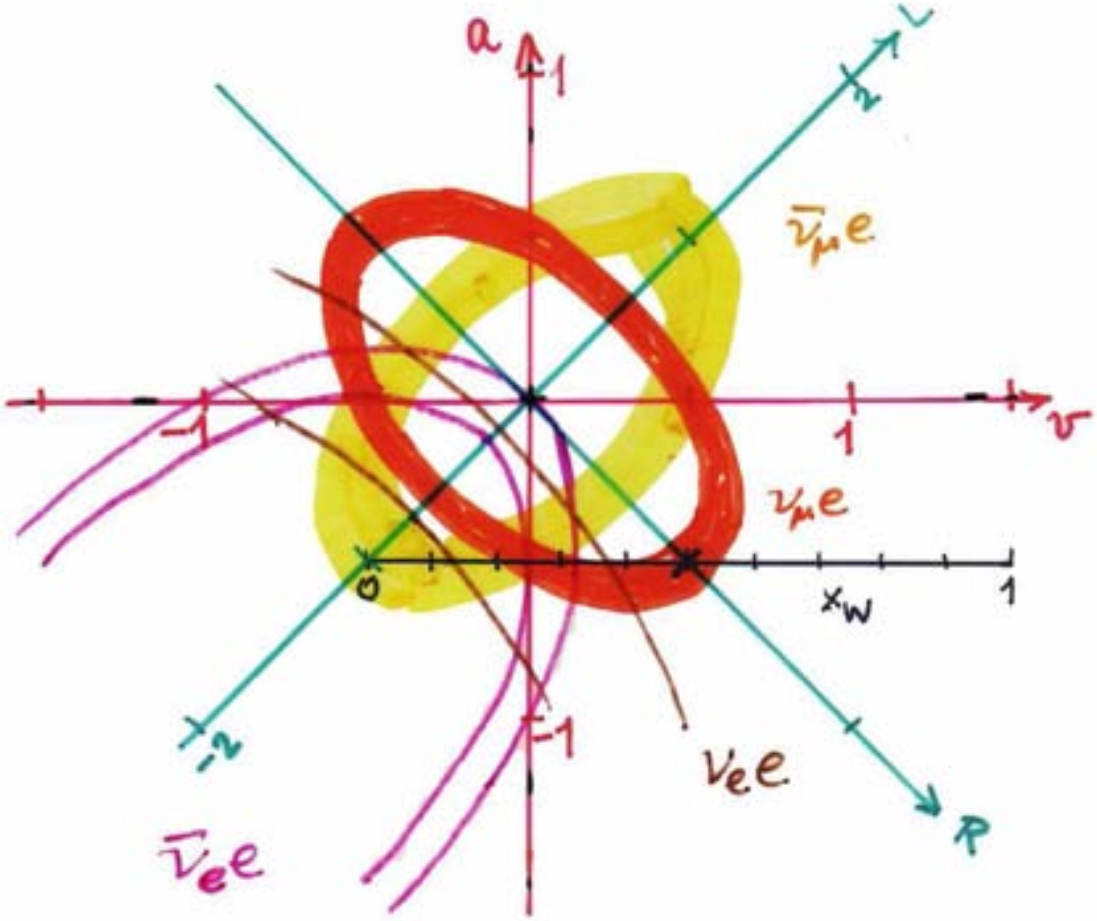
Measure all cross sections to determine chiral couplings L_e and R_e or traditional vector and axial couplings v and a

$$a = \frac{1}{2}(L_e - R_e) \qquad v = \frac{1}{2}(L_e - R_e)$$

$$L_e = v + a \qquad R_e = v - a$$

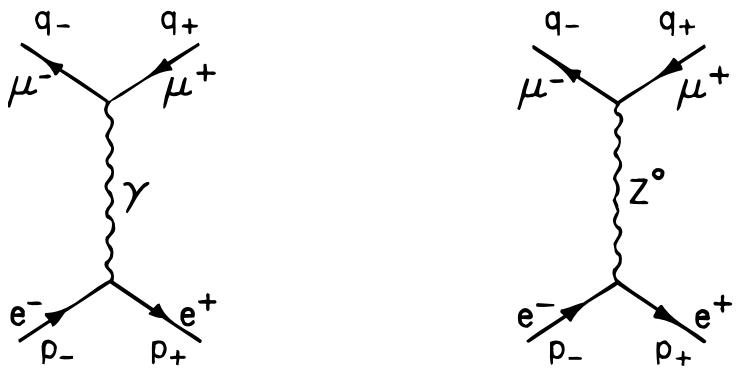
model-independent in V, A framework

Neutrino–electron scattering



Twofold ambiguity remains even after measuring all four cross sections: same cross sections result if we interchange $R_e \leftrightarrow -R_e$ $(v \leftrightarrow a)$

Consider $e^+e^- \to \mu^+\mu^-$



$$
\begin{aligned}
\mathcal{M} \;=\; & -ie^2\bar{u}(\mu,q_-)\gamma_\lambda Q_\mu v(\mu,q_+)\frac{g^{\lambda\nu}}{s}\bar{v}(e,p_+)\gamma_\nu u(e.p_-) \\
& +\frac{i}{2}\left(\frac{G_F M_Z^2}{\sqrt{2}}\right)\bar{u}(\mu,q_-)\gamma_\lambda[R_\mu(1+\gamma_5)+L_\mu(1-\gamma_5)]v(\mu,q_+) \\
& \times\frac{g^{\lambda\nu}}{s-M_Z^2}\bar{v}(e,p_+)\gamma_\nu[R_e(1+\gamma_5)+L_e(1-\gamma_5)]u(e,p_-)
\end{aligned}
$$

muon charge $Q_\mu = -1$

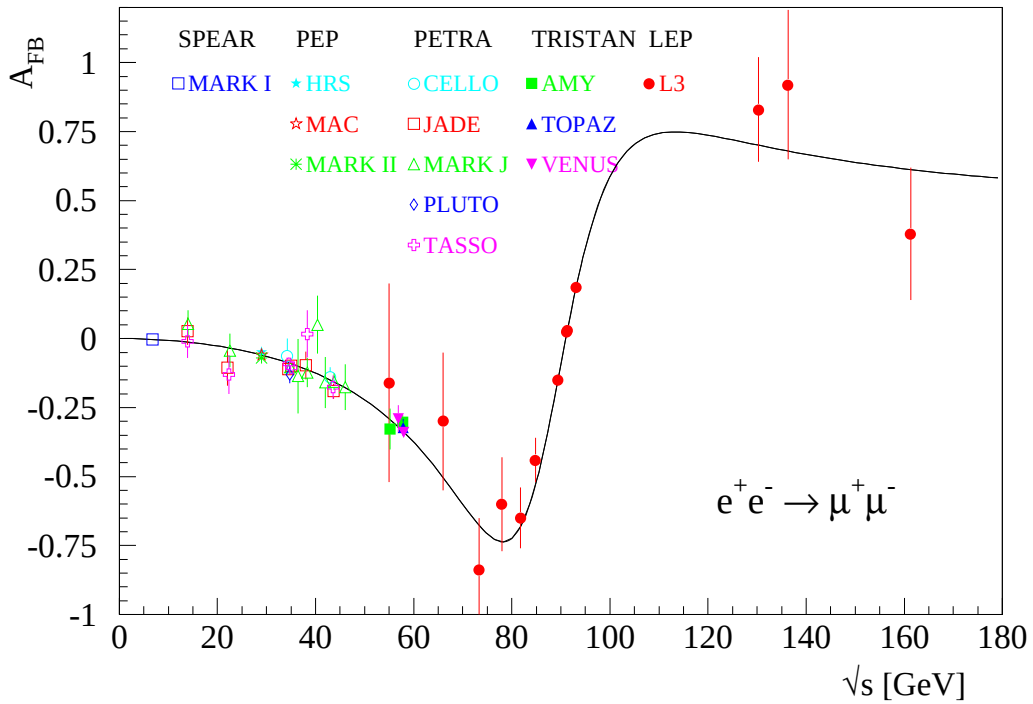
$e^+e^- \to \mu^+\mu^- \ldots$

$$
\begin{aligned}
\frac{d\sigma}{dz} &= \frac{\pi\alpha^2 Q_\mu^2}{2s}(1+z^2) \\
&\quad - \frac{\alpha Q_\mu G_F M_Z^2 (s - M_Z^2)}{8\sqrt{2}[(s-M_Z^2)^2 + M_Z^2\Gamma^2]} \\
&\quad \times [(R_e + L_e)(R_\mu + L_\mu)(1+z^2) + 2(R_e - L_e)(R_\mu - L_\mu)z] \\
&\quad + \frac{G_F^2 M_Z^4 s}{64\pi[(s-M_Z^2)^2 + M_Z^2\Gamma^2]} \\
&\quad \times [(R_e^2 + L_e^2)(R_\mu^2 + L_\mu^2)(1+z^2) + 2(R_e^2 - L_e^2)(R_\mu^2 - L_\mu^2)z]
\end{aligned}
$$

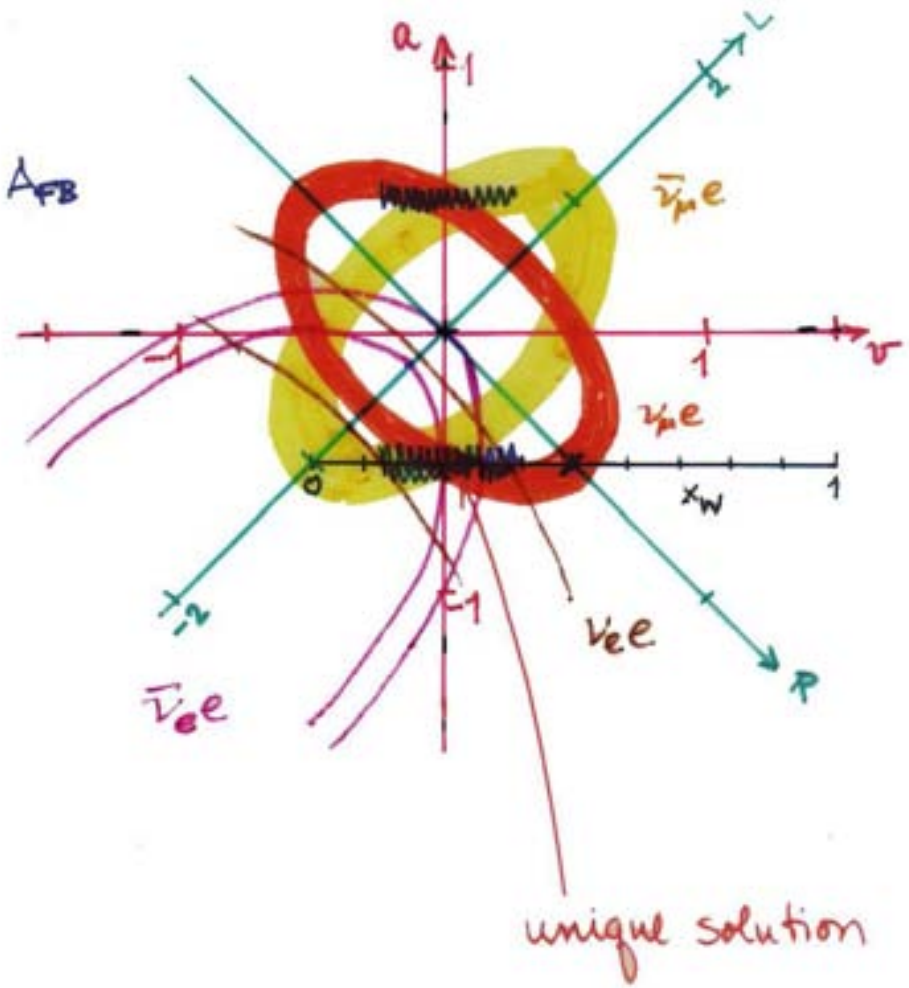
Measuring A resolves ambiguity

$$\text{Forward-backward asymmetry } A \equiv \frac{\int_0^1 dz\, d\sigma/dz - \int_{-1}^0 dz\, d\sigma/dz}{\int_{-1}^1 dz\, d\sigma/dz}$$

$$\begin{aligned} \lim_{s/M_Z^2 \ll 1} A &= \frac{3G_F s}{16\pi\alpha Q_\mu \sqrt{2}}(R_e - L_e)(R_\mu - L_\mu) \\ &\approx -6.7 \times 10^{-5} \left(\frac{s}{1\ \text{GeV}^2}\right)(R_e - L_e)(R_\mu - L_\mu) = -3G_F s a^2/4\pi\alpha\sqrt{2} \end{aligned}$$



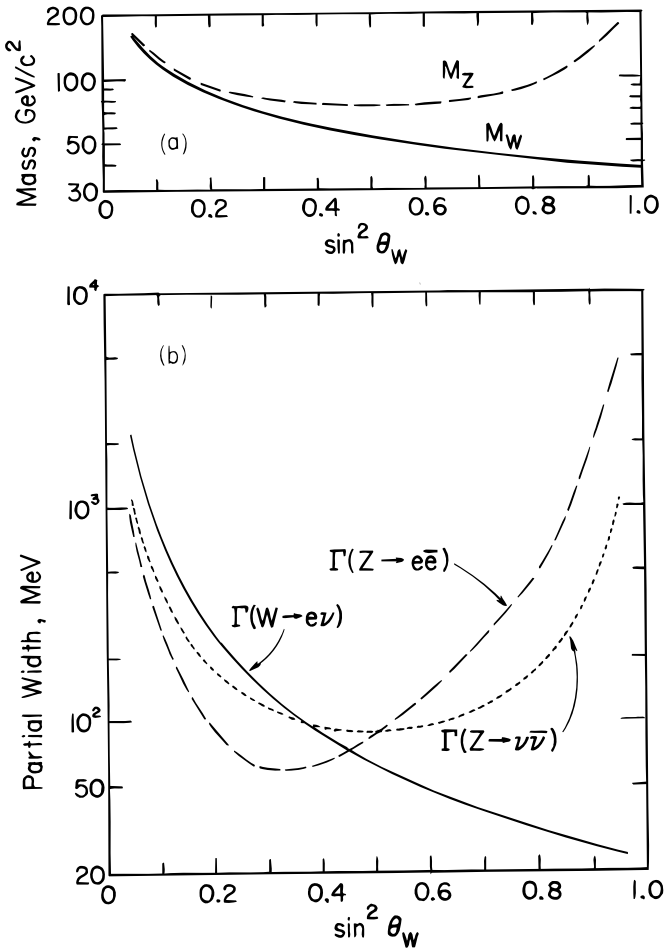
Neutrino–electron scattering $e^+e^- \to \mu^+\mu^-$



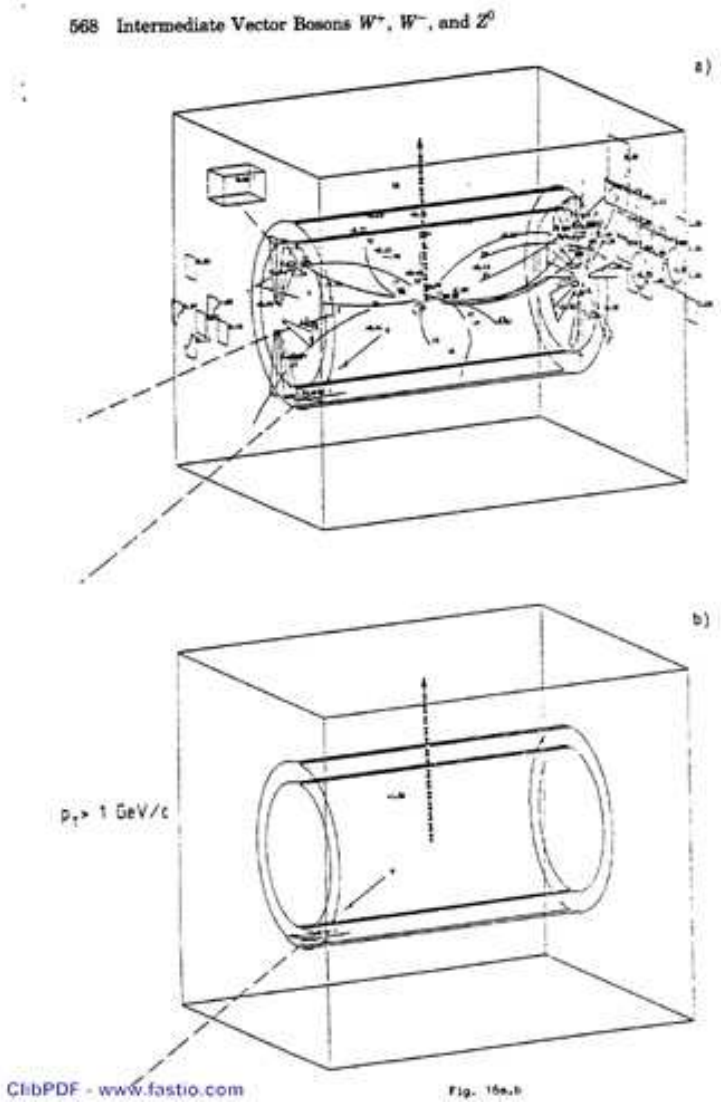
Validate EW theory, measure $\sin^2\theta_W$

With a measurement of $\sin^2\theta_W$, predict

$$M_W^2 = \pi\alpha / G_F\sqrt{2}\sin^2\theta_W \approx (37.28\ \mathrm{GeV}/c^2)^2/\sin^2\theta_W \qquad M_Z^2 = M_W^2/\cos^2\theta_W$$



First Z from UA1



Electroweak interactions of quarks (one generation)

- Left-handed doublet

$$\mathrm{L}_q = \begin{pmatrix} u \\ d \end{pmatrix}_L \qquad \begin{array}{ccc} I_3 & Q & Y = 2(Q - I_3) \\ \frac{1}{2} & +\frac{2}{3} & \\ -\frac{1}{2} & -\frac{1}{3} & \frac{1}{3} \end{array}$$

- two right-handed singlets

$$\begin{array}{cccc} & I_3 & Q & Y = 2(Q - I_3) \\ \mathrm{R}_u = u_R & 0 & +\frac{2}{3} & +\frac{4}{3} \\ \mathrm{R}_d = d_R & 0 & -\frac{1}{3} & -\frac{2}{3} \end{array}$$

Electroweak interactions of quarks

- CC interaction

$$\mathcal{L}_{W\text{-}q} = \frac{-g}{2\sqrt{2}} \left[\bar{u}_e \gamma^\mu (1-\gamma_5) d \; W_\mu^+ + \bar{d} \gamma^\mu (1-\gamma_5) u \; W_\mu^- \right]$$

identical in form to $\mathcal{L}_{W\text{-}\ell}$: universality $\Leftrightarrow$ weak isospin

- NC interaction

$$\mathcal{L}_{Z\text{-}q} = \frac{-g}{4\cos\theta_W} \sum_{i=u,d} \bar{q}_i \gamma^\mu \left[L_i (1-\gamma_5) + R_i (1+\gamma_5) \right] q_i \; Z_\mu$$

$$L_i = \tau_3 - 2Q_i \sin^2\theta_W \quad R_i = -2Q_i \sin^2\theta_W$$

equivalent in form (not numbers) to $\mathcal{L}_{Z\text{-}\ell}$

Trouble in Paradise

Universal $u \leftrightarrow d$, $\nu_e \leftrightarrow e$ *not quite right*

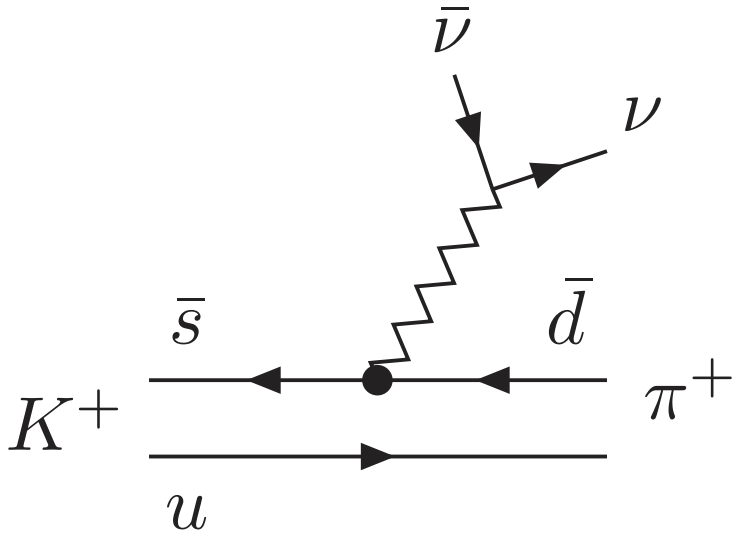
$$\text{Good:} \quad \begin{pmatrix} u \\ d \end{pmatrix}_L \rightarrow \text{Better:} \quad \begin{pmatrix} u \\ d_\theta \end{pmatrix}_L$$

$$d_\theta \equiv d \, \cos\theta_C + s \, \sin\theta_C \qquad \cos\theta_C = 0.9736 \pm 0.0010$$

"Cabibbo-rotated" doublet perfects CC interaction (up to small third-generation effects) but $\Rightarrow$ serious trouble for NC

$$\begin{aligned}
\mathcal{L}_{Z\text{-}q} \;=\; & \frac{-g}{4\cos\theta_W} Z_\mu \{ \bar{u}\gamma^\mu \left[L_u(1-\gamma_5) + R_u(1+\gamma_5) \right] u \\
& + \bar{d}\gamma^\mu \left[L_d(1-\gamma_5) + R_d(1+\gamma_5) \right] d \, \cos^2\theta_C \\
& + \bar{s}\gamma^\mu \left[L_d(1-\gamma_5) + R_d(1+\gamma_5) \right] s \, \sin^2\theta_C \\
& + \bar{d}\gamma^\mu \left[L_d(1-\gamma_5) + R_d(1+\gamma_5) \right] s \, \sin\theta_C \cos\theta_C \\
& + \bar{s}\gamma^\mu \left[L_d(1-\gamma_5) + R_d(1+\gamma_5) \right] d \, \sin\theta_C \cos\theta_C \}
\end{aligned}$$

Strangeness-changing NC interactions highly suppressed!



BNL E-787/E-949 has three $K^+ \to \pi^+ \nu\bar{\nu}$ candidates, with $\mathcal{B}(K^+ \to \pi^+ \nu\bar{\nu}) = 1.5^{+1.3}_{-0.9} \times 10^{-10}$ *Phys. Rev. Lett.* **93**, 031801 (2004)

(SM: 0.78 ± 0.11: U. Haisch, hep-ph/0605170)

Glashow–Iliopoulos–Maiani

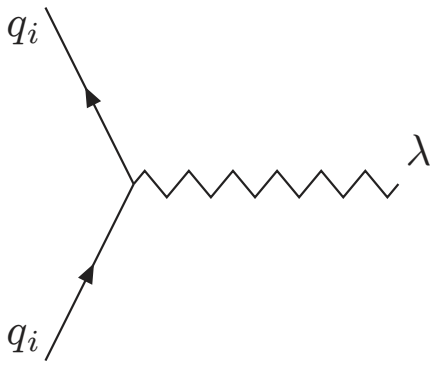
two LH doublets: $\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \quad \begin{pmatrix} u \\ d_\theta \end{pmatrix}_L \begin{pmatrix} c \\ s_\theta \end{pmatrix}_L$

$$(s_\theta = s\,\cos\theta_C - d\,\sin\theta_C)$$

+ right-handed singlets, e_R, μ_R, u_R, d_R, c_R, s_R

Required new charmed quark, c

Cross terms vanish in $\mathcal{L}_{Z\text{-}q}$,



$$\frac{-ig}{4\cos\theta_W}\gamma_\lambda[(1-\gamma_5)L_i + (1+\gamma_5)R_i] \quad ,$$

$$L_i = \tau_3 - 2Q_i \sin^2\theta_W \quad R_i = -2Q_i \sin^2\theta_W$$

flavor-diagonal interaction!

Straightforward generalization to n quark doublets

$$\mathcal{L}_{W\text{-}q} = \frac{-g}{2\sqrt{2}} \left[\bar{\Psi} \gamma^\mu (1-\gamma_5) \mathcal{O} \Psi \, W_\mu^+ + \text{h.c.} \right]$$

$$\text{composite } \Psi = \begin{pmatrix} u \\ c \\ \vdots \\ \\ d \\ s \\ \vdots \end{pmatrix}$$

flavor structure $\mathcal{O} = \begin{pmatrix} 0 & U \\ 0 & 0 \end{pmatrix}$

U: unitary quark mixing matrix

Weak-isospin part: $\mathcal{L}_{Z\text{-}q}^{\text{iso}} = \dfrac{-g}{4\cos\theta_W} \bar{\Psi} \gamma^\mu (1-\gamma_5) \left[\mathcal{O}, \mathcal{O}^\dagger \right] \Psi$

Since $\left[\mathcal{O}, \mathcal{O}^\dagger \right] = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \propto \tau_3$

$\boxed{\Rightarrow \text{NC interaction is flavor-diagonal}}$

General $n \times n$ mixing matrix U: $n(n-1)/2$ real $\angle$, $(n-1)(n-2)/2$ complex phases

3×3 (Cabibbo–Kobayashi-Maskawa): $3 \angle + 1$ phase $\Rightarrow \mathcal{CP}$ violation

Successful predictions of $SU(2)_L \otimes U(1)_Y$ theory:

- neutral-current interactions
- necessity of charm
- existence and properties of $W^\pm$ and Z^0

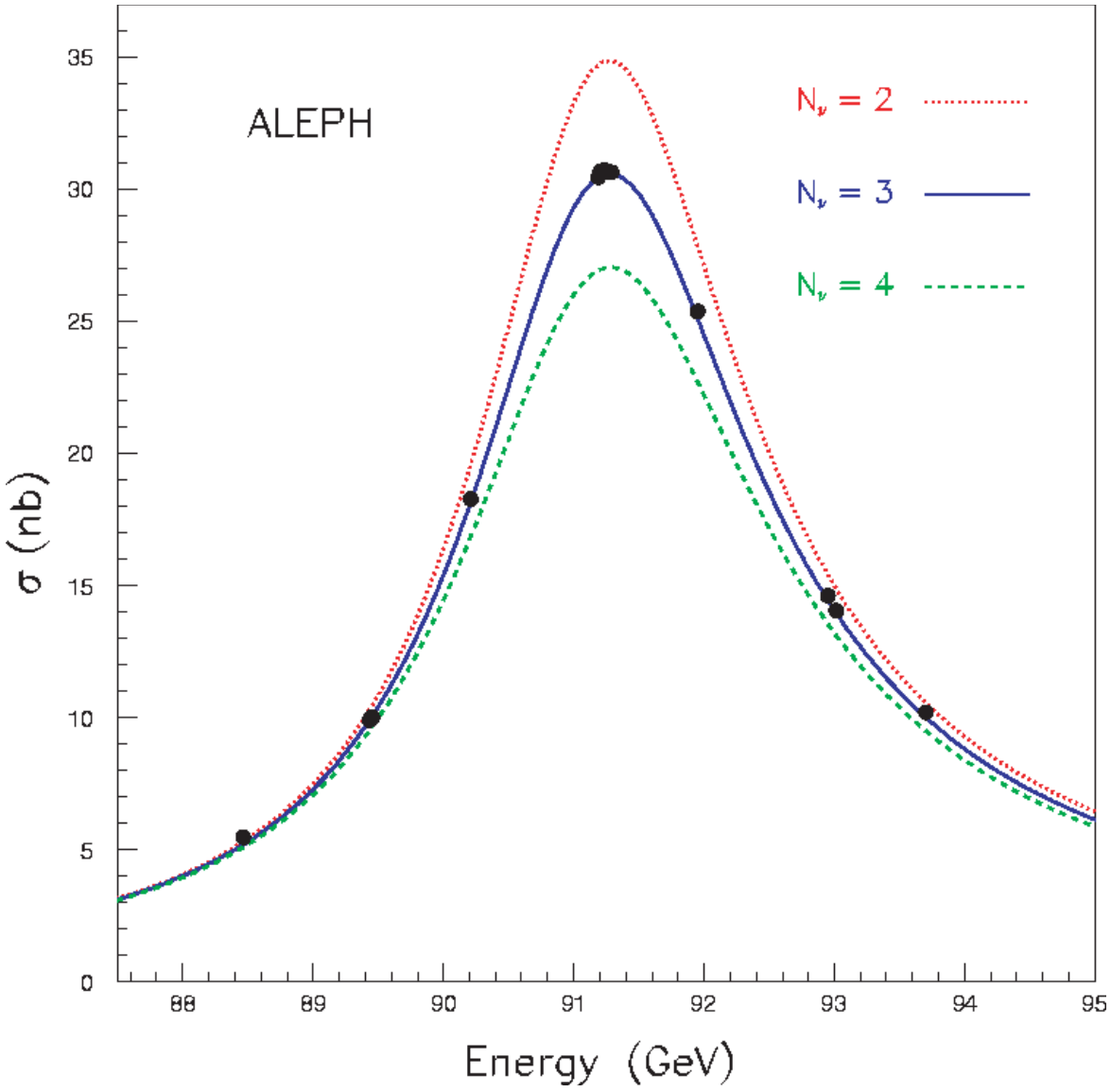
+ a decade of precision EW tests (one-per-mille)

M_W	$80\,398 \pm 25$ MeV
Γ_W	$2\,140 \pm 60$ MeV
M_Z	$91\,187.6 \pm 2.1$ MeV
Γ_Z	2495.2 ± 2.3 MeV
$\sigma^0_{\text{hadronic}}$	41.541 ± 0.037 nb
Γ_{hadronic}	1744.4 ± 2.0 MeV
Γ_{leptonic}	83.984 ± 0.086 MeV
$\Gamma_{\text{invisible}}$	499.0 ± 1.5 MeV

$$\Gamma_{\text{invisible}} \equiv \Gamma_Z - \Gamma_{\text{hadronic}} - 3\Gamma_{\text{leptonic}}$$

$$\text{light } \nu : N_\nu = \Gamma_{\text{invisible}}/\Gamma^{\text{SM}}(Z \to \nu_i \bar{\nu}_i) = 2.994 \pm 0.012 \quad (\nu_e, \nu_\mu, \nu_\tau)$$

Three light neutrinos



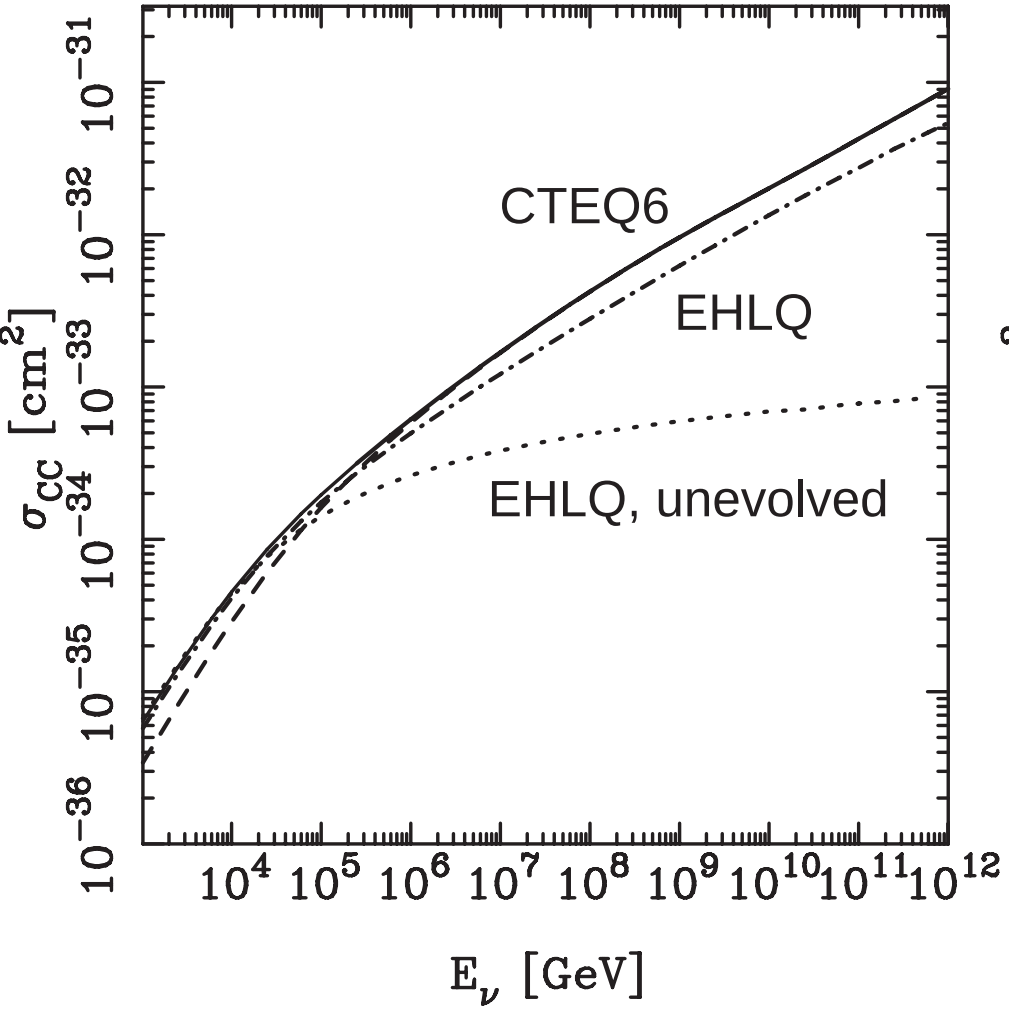
$\nu_\mu N \to \mu^- +$ anything: *influence of W propagator*

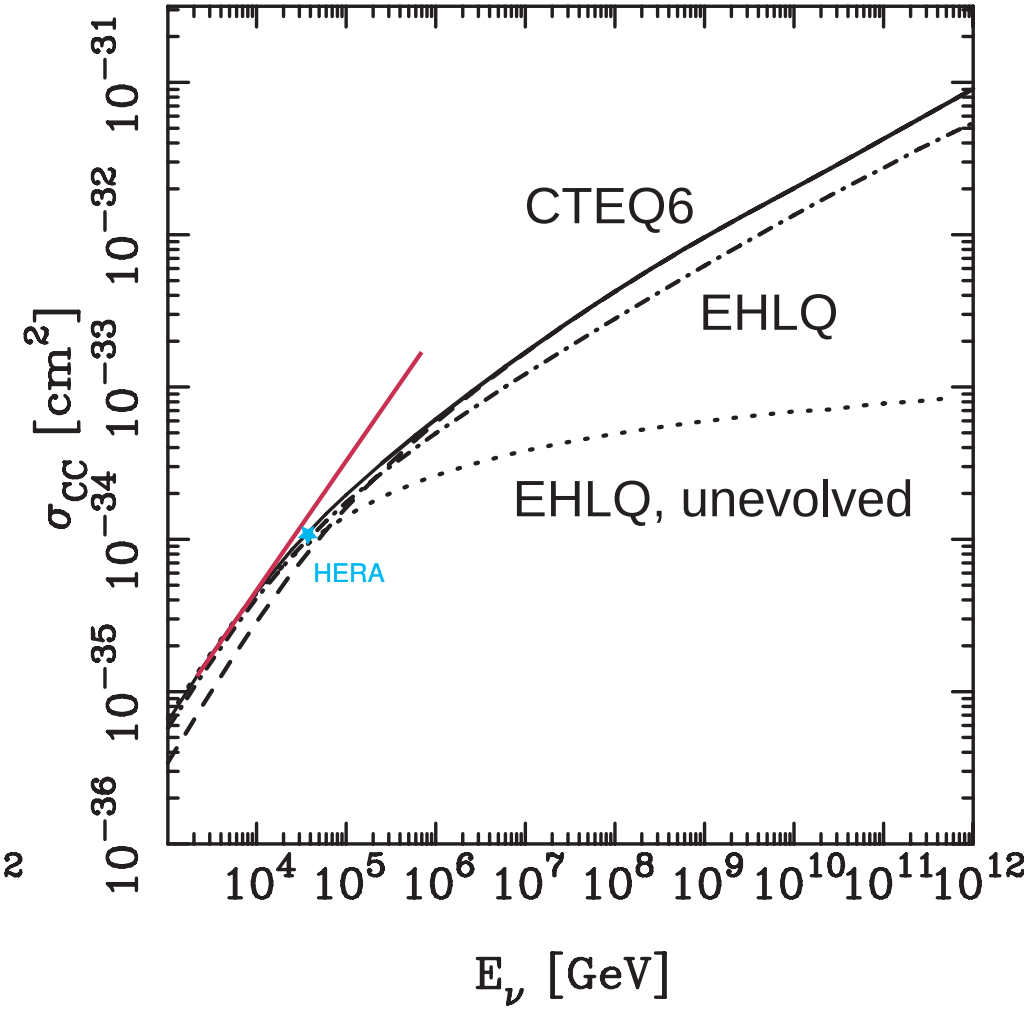
$$\frac{d^2\sigma}{dxdy} = \frac{2G_F^2 M E_\nu}{\pi}\left(\frac{M_W^2}{Q^2+M_W^2}\right)^2 \left[xq(x,Q^2) + x\bar{q}(x,Q^2)(1-y)^2\right]$$

$$\begin{aligned} q(x,Q^2) &= \frac{u_v(x,Q^2)+d_v(x,Q^2)}{2} + \frac{u_s(x,Q^2)+d_s(x,Q^2)}{2} \\ &\quad + s_s(x,Q^2) + b_s(x,Q^2) \\ \bar{q}(x,Q^2) &= \frac{u_s(x,Q^2)+d_s(x,Q^2)}{2} + c_s(x,Q^2) + t_s(x,Q^2), \end{aligned}$$

. . . isoscalar nucleon

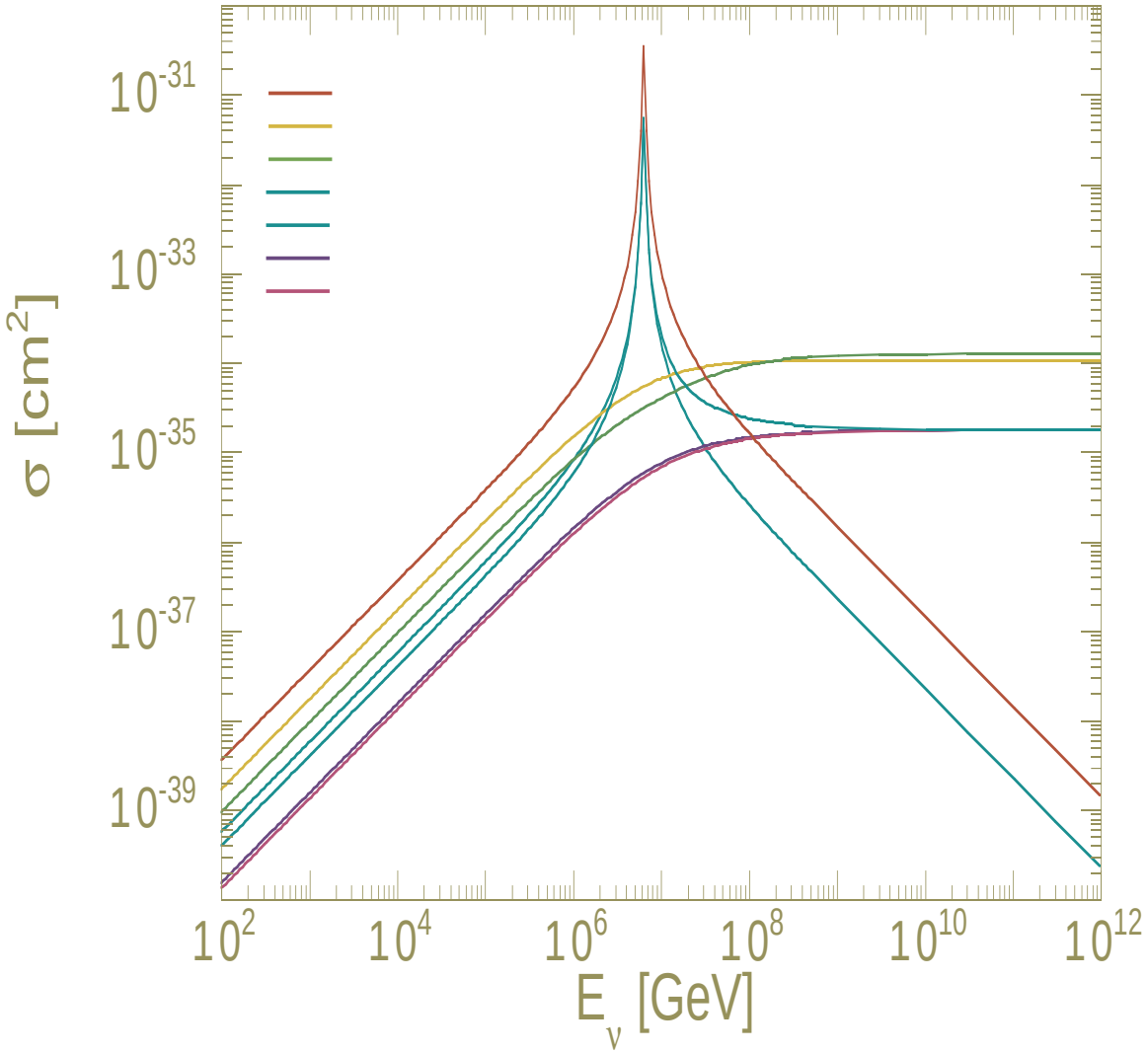
$$x = Q^2/2M\nu \qquad y \equiv \nu/E_\nu \qquad \nu \equiv E_\nu - E_\mu$$


CTEQ6
EHLQ
EHLQ, unevolved
σ_{CC} [cm^2]
E_ν [GeV]


CTEQ6
EHLQ
EHLQ, unevolved
HERA
σ_{CC} [cm^2]
E_ν [GeV]

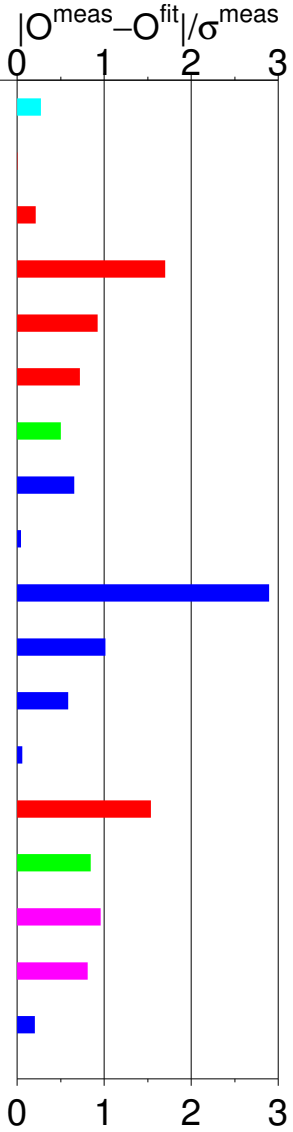
M. H. Reno

νe cross sections . . .



At low energies: $\sigma(\bar{\nu}_e e \to$ hadrons$) > \sigma(\nu_\mu e \to \mu\nu_e) > \sigma(\nu_e e \to \nu_e e) > \sigma((\bar{\nu}_e e \to \bar{\nu}_\mu \mu) > \sigma(\bar{\nu}_e e \to \bar{\nu}_e e) > \sigma(\nu_\mu e \to \nu_\mu e) > \sigma(\bar{\nu}_\mu e \to \bar{\nu}_\mu e)$

	Measurement	Fit
$\Delta\alpha^{(5)}_{had}(m_Z)$	0.02758 ± 0.00035	0.02768
m_Z [GeV]	91.1875 ± 0.0021	91.1875
Γ_Z [GeV]	2.4952 ± 0.0023	2.4957
σ^0_{had} [nb]	41.540 ± 0.037	41.477
R_l	20.767 ± 0.025	20.744
$A^{0,l}_{fb}$	0.01714 ± 0.00095	0.01645
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1481
R_b	0.21629 ± 0.00066	0.21586
R_c	0.1721 ± 0.0030	0.1722
$A^{0,b}_{fb}$	0.0992 ± 0.0016	0.1038
$A^{0,c}_{fb}$	0.0707 ± 0.0035	0.0742
A_b	0.923 ± 0.020	0.935
A_c	0.670 ± 0.027	0.668
A_l(SLD)	0.1513 ± 0.0021	0.1481
$\sin^2\theta^{lept}_{eff}(Q_{fb})$	0.2324 ± 0.0012	0.2314
m_W [GeV]	80.398 ± 0.025	80.374
Γ_W [GeV]	2.140 ± 0.060	2.091
m_t [GeV]	170.9 ± 1.8	171.3

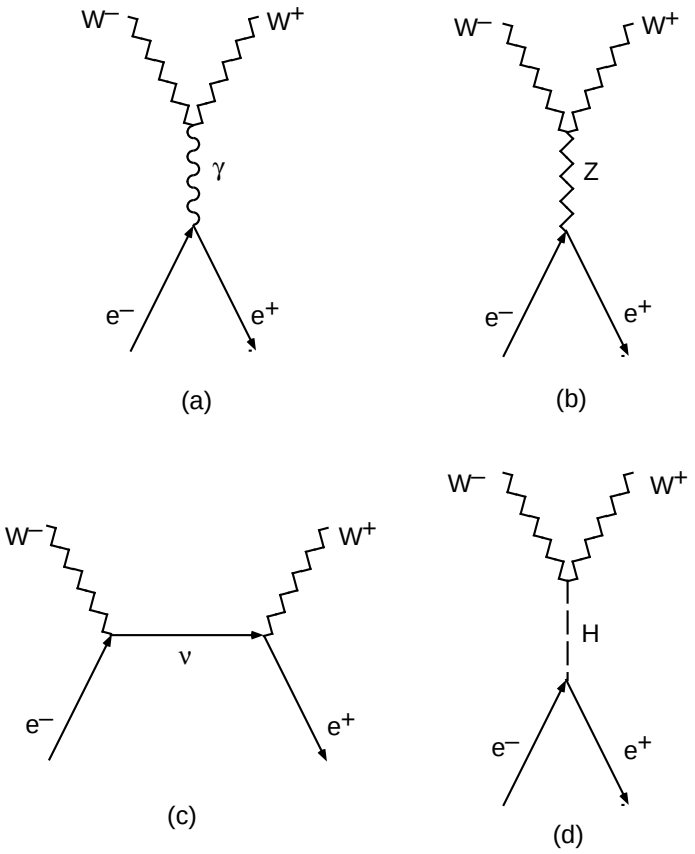


LEP Electroweak Working Group, Winter 2007

Why a Higgs boson must exist

▷ Role in canceling high-energy divergences

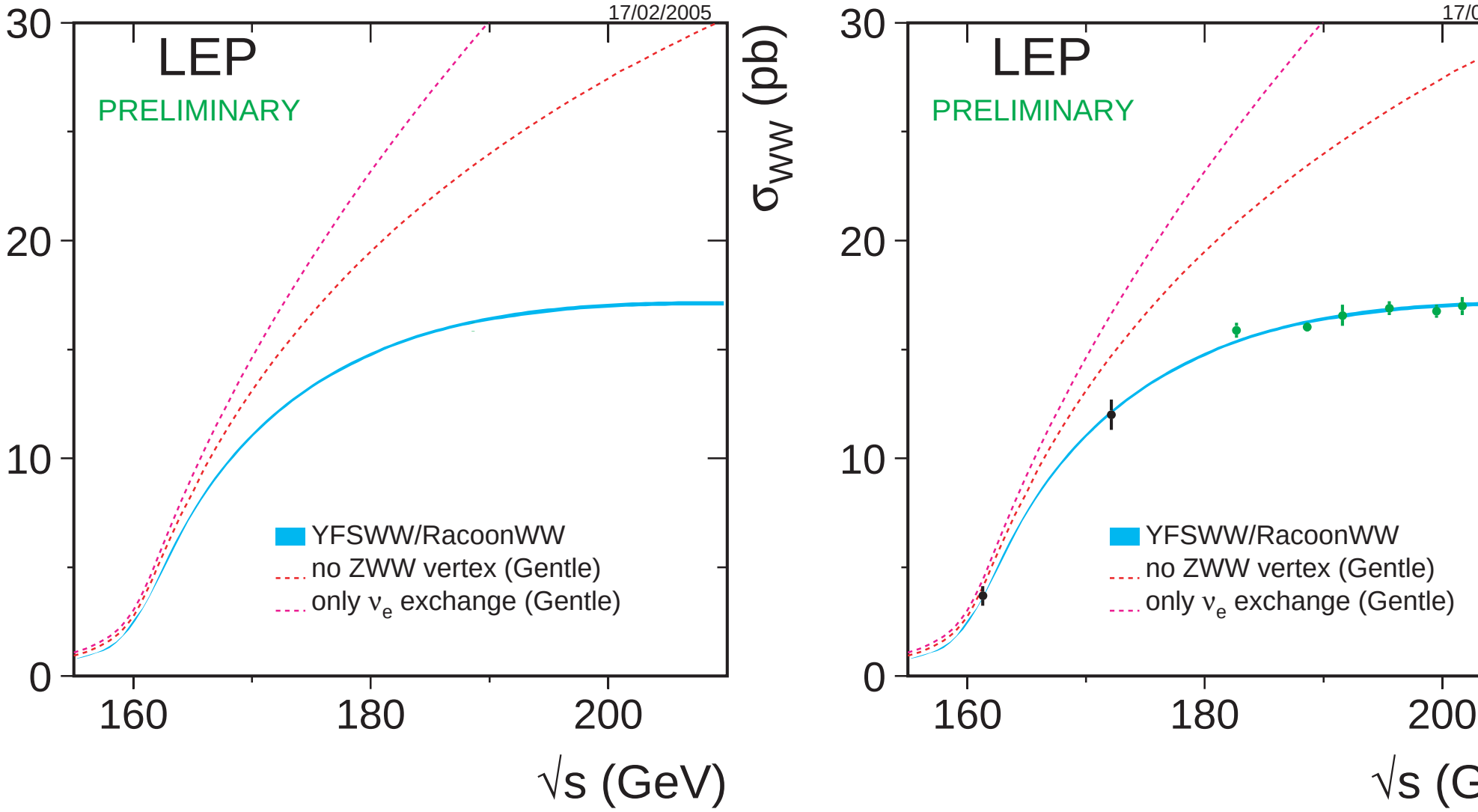
S-matrix analysis of $e^+e^- \to W^+W^-$



Individual $J = 1$ partial-wave amplitudes $\mathcal{M}_\gamma^{(1)}$, $\mathcal{M}_Z^{(1)}$, $\mathcal{M}_\nu^{(1)}$ have unacceptable high-energy behavior $(\propto s)$

. . . But sum is well-behaved

"Gauge cancellation" observed at LEP2 (Tevatron)



$J = 0$ amplitude exists because electrons have mass, and can be found in "wrong" helicity state

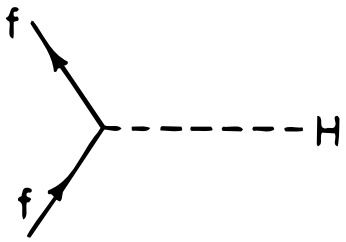
$$\mathcal{M}_\nu^{(0)} \propto s^{\frac{1}{2}} \quad \text{: unacceptable HE behavior}$$

(no contributions from γ and Z)

This divergence is canceled by the Higgs-boson contribution

$$\boxed{\Rightarrow He\bar{e} \text{ coupling must be } \propto m_e,}$$

because "wrong-helicity" amplitudes $\propto m_e$



$$\frac{-im_f}{v} = -im_f(G_F\sqrt{2})^{1/2}$$

If the Higgs boson did not exist, something else would have to cure divergent behavior

If gauge symmetry were unbroken . . .

- no Higgs boson
- no longitudinal gauge bosons
- no extreme divergences
- no wrong-helicity amplitudes

. . . and no viable low-energy phenomenology

In spontaneously broken theory . . .

- gauge structure of couplings eliminates the most severe divergences
- lesser—but potentially fatal—divergence arises because the electron has mass . . . due to the Higgs mechanism
- SSB provides its own cure—the Higgs boson

Similar interplay & compensation *must exist* in any acceptable theory