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Neutrino Cross Sections

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Outline

1. Introduction
   1. Neutrino Oscillation Experiments
   2. Role of neutrino cross sections
   3. $\nu+e \rightarrow \nu+e$ scattering
2. Cross Sections of ~MeV neutrinos
3. Cross Sections of ~GeV neutrinos
4. Cross Sections of 10~100GeV neutrinos
5. Cross Sections of >TeV neutrinos
6. Experiments for neutrino cross sections.
7. Summary

Revolution of Neutrino Physics

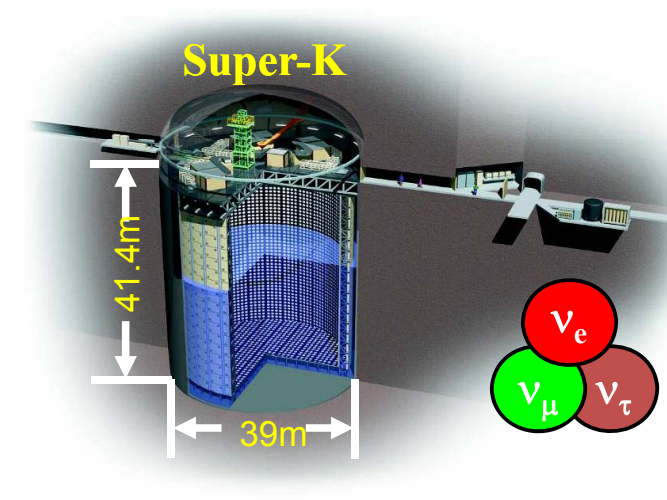
- Discovery of Neutrino mass.
- Discovery of Neutrino Mixing.

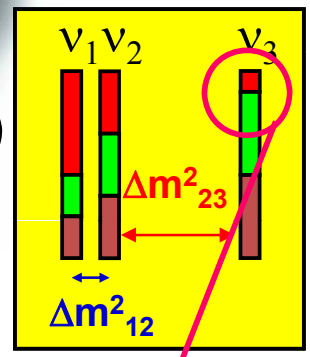
- Unresolved Problems!
  - Neutrino Mass Type
  - Neutrino Mixing Scheme (3 generation)
  - CP Violation
  - Sterile Neutrinos
  - Non-standard ν interactions?
  -

Neutrino Oscillation Experiments are important!!

1. Introduction-1

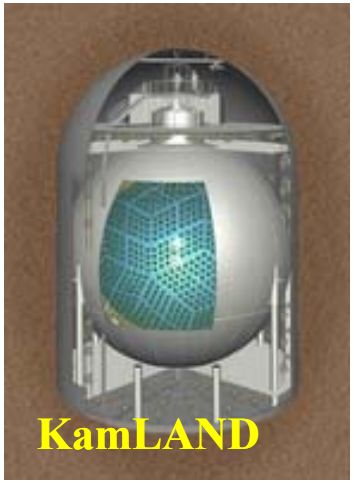
Neutrino Oscillation Experiments.

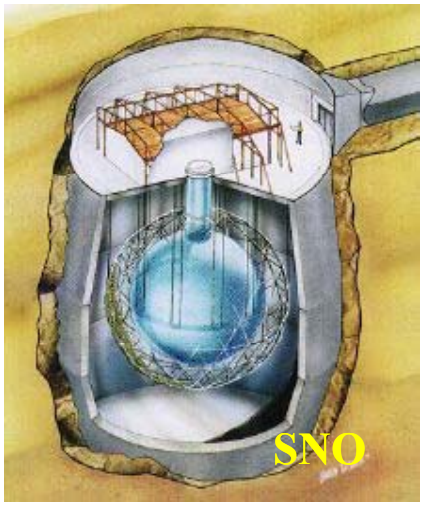




Neutrino masses **(Δm_{12}^2, Δm_{23}^2)**
Mixing Angles **(θ_{12}, θ_{23})**

$\theta_{13} \rightarrow \delta$





1. Introduction (1.1)

Neutrino Oscillation Experiments.

- Experimental condition:
  - Solar Neutrinos (__)

    E_ν~_____, L~_____, Δm^2 ~_____, σ_ν ~_____.
  - Reactor Neutrinos (__)

    E_ν~_____, L~_____, Δm^2 ~_____, σ_ν ~_____.
  - Atmospheric Neutrinos (__, __, __, __)

    E_ν~_____, L~_____, Δm^2 ~_____, σ_ν ~_____.
  - Accelerator Neutrinos (__, __, __, __)

    E_ν~_____, L~_____, Δm^2 ~_____, σ_ν ~_____.

- Neutrino Targets:
  - Solar Neutrinos:
    - Gallex(GNO), SAGE, Homesatke, Kamiokande, Super-K, SNO

    _____,
  - Reactor Neutrinos:
    - CHOOZ, KamLAND

    _____,
  - Atmospheric Neutrinos:
    - Soudan2, Kamioaknde, Super-K, MINOS, MACRO

    _____,
  - Accelerator Neutrinos:
    - LSND, MiniBooNE, K2K, MINOS, OPERA, CCFR, CHORUS, NOMAD

    _____,

- Neutrino Flux:
  - Solar Neutrinos:
  - Reactor Neutrinos:
  - Atmospheric Neutrinos:
  - Accelerator Neutrinos:

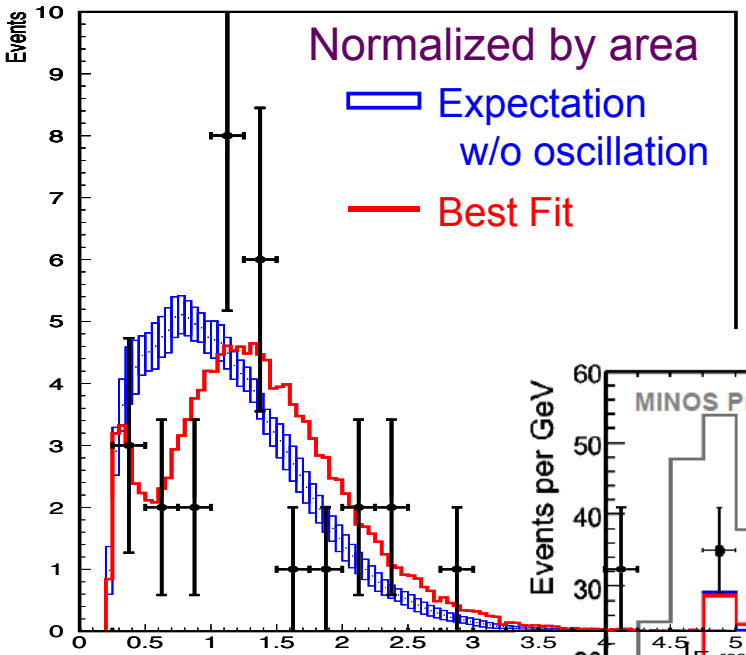
HOMEWORK: See the reference.

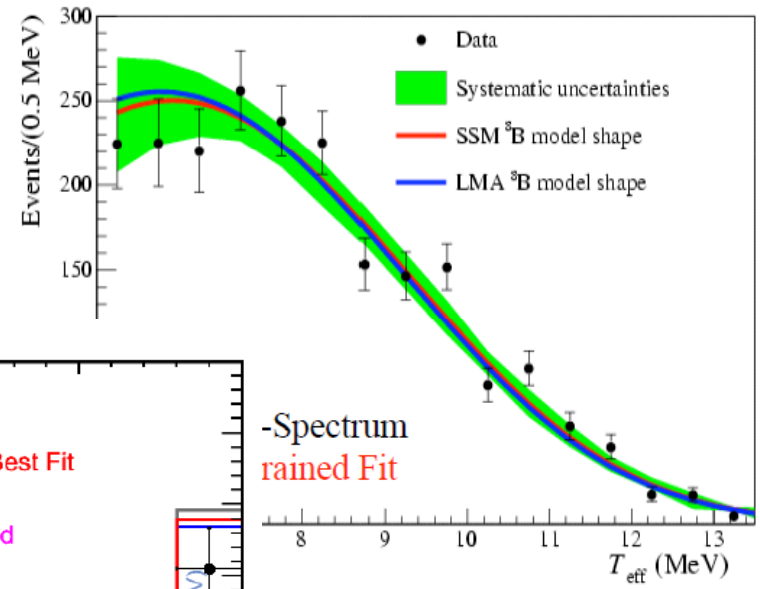
1. Introduction-2

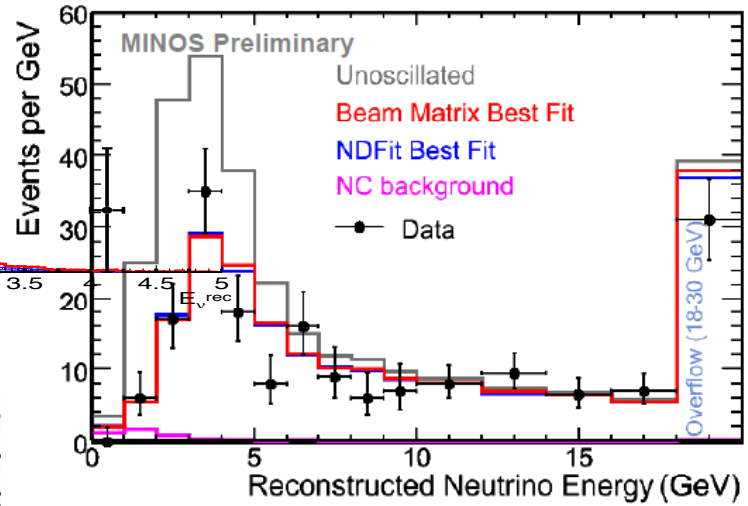
- Role of Neutrino Cross Sections
  - In order to understand the results of neutrino oscillations including experimental designs and methods, the understanding of neutrino interaction (cross section) is essential.
  - For experimentalists
    - It is essential for you to design an experiment and get reliable results.
  - For theorists
    - The neutrino cross section is often used to estimate the systematic error on the sensitivity of neutrino oscillation.

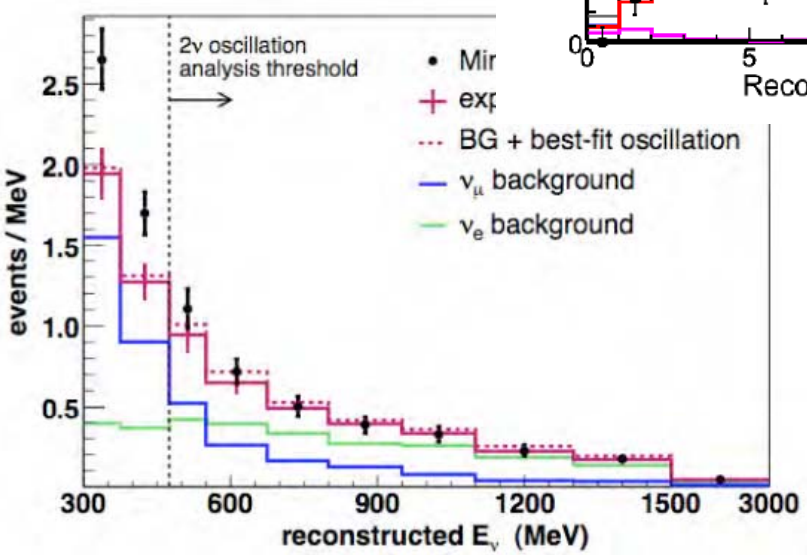
Necessary information for oscillation study:

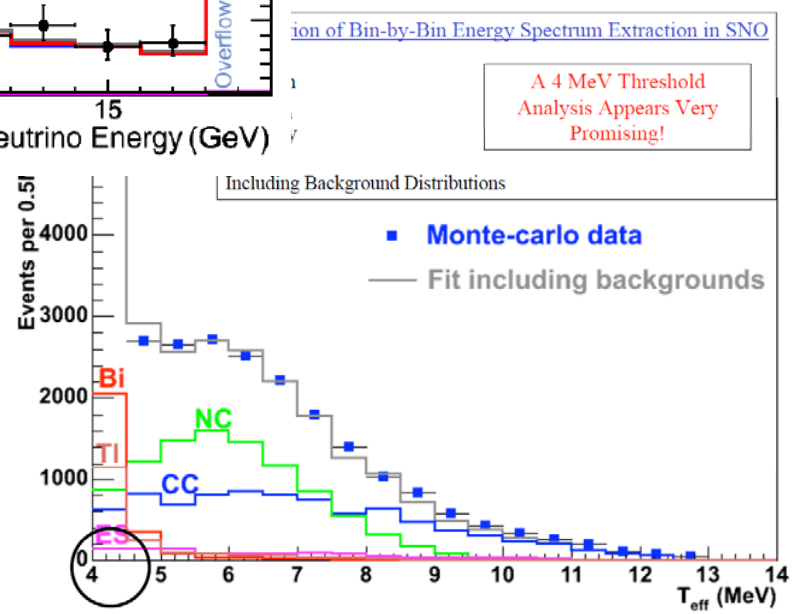
- Neutrino flux measurement
  - Cross section
- Neutrino energy measurement (reconstruction)
  - Interaction modes with the cross sections.
    - What is the target?
    - Kinematics
    - How interaction modes will vary as a function of neutrino energy?
- Background estimate (& Systematic errors)
  - Interaction modes with the cross sections.
    - What is the target?
    - Kinematics
    - How interaction modes will vary as a function of neutrino energy?


Normalized by area
Expectation
w/o oscillation
Best Fit
Events


Data
Systematic uncertainties
SSM ^{8}B model shape
LMA ^{8}B model shape
Events/(0.5 MeV)
-Spectrum
rained Fit
T_{eff} (MeV)


MINOS Preliminary
Unoscillated
Beam Matrix Best Fit
NDFit Best Fit
NC background
Data
Events per GeV
Overflow (18-30 GeV)
Reconstructed Neutrino Energy (GeV)


2ν oscillation
analysis threshold
BG + best-fit oscillation
ν_μ background
ν_e background
events / MeV
reconstructed E_ν (MeV)


ion of Bin-by-Bin Energy Spectrum Extraction in SNO
A 4 MeV Threshold
Analysis Appears Very
Promising!
Including Background Distributions
Monte-carlo data
Fit including backgrounds
Events per 0.5
Bi
Tl
NC
CC
ES
T_{eff} (MeV)

- Keep the followings in your mind.
  - Many types of neutrino targets, and they look different with the neutrino energy.
    - Electron
    - quark
    - Proton and Neutron
    - Nucleus
  - Charged current and Neutral Current Interactions.

Because of the above complexity, the neutrino cross section looks not easy for non-experts!

Let's learn the physics of neutrino cross sections from the basic in this lecture!!

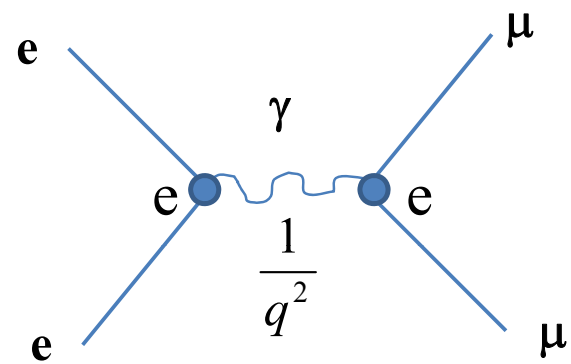
1. Introduction-3

ν+e → ν+e scattering

- Let's review the QED process first,

  assuming m_ν, m_e,m_μ=0:

  – $e^+e^- \rightarrow \mu^+\mu^-$



$$M_{if} = \frac{4\pi\alpha}{q^2}$$

$$\frac{d\sigma}{d\Omega}(e^+e^- \rightarrow \mu^+\mu^-) = \frac{\alpha^2}{4s}(1+\cos^2\theta) = \frac{\alpha^2}{8p^2}\left(\frac{t^2+u^2}{s^2}\right)$$

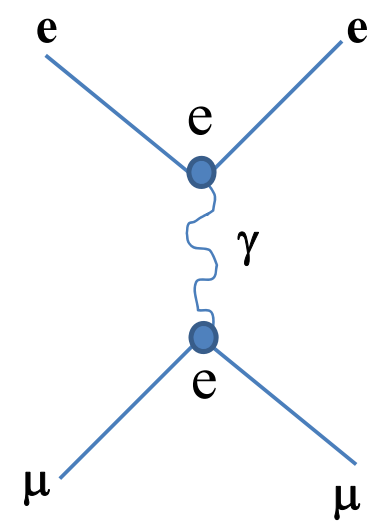
$$\sigma(e^+e^- \rightarrow \mu^+\mu^-) = \frac{4\pi\alpha^2}{3s}$$

- Review the QED process,
  - $e^-\mu^+ \rightarrow e^-\mu^+$; Text example (not real process)

$s \rightarrow -t$

$$\frac{d\sigma}{d\Omega}(e^-\mu^+ \rightarrow e^-\mu^+) = \frac{\alpha^2}{8p^2}\left(\frac{s^2+u^2}{t^2}\right)$$

$$= \frac{\alpha^2}{8p^2\sin^4(\theta/2)}\left[1+\cos^4(\theta/2)\right]$$



Lab frame (muon rest frame)

$$E'_\mu = \gamma p(1-\cos\theta), E_e = 2\gamma p, y = \frac{E'_\mu}{E_e} = \frac{1-\cos\theta}{2}$$

$$\cos^2\frac{\theta}{2} = \frac{1+\cos\theta}{2} = 1-y$$

$$d\Omega = 2\pi d(\cos\theta) = 4\pi dy$$

$$\frac{d\sigma}{dy}(e^-\mu^+ \rightarrow e^-\mu^+) = \frac{2\pi\alpha^2 s}{q^4}\left[1+(1-y)^2\right]$$

$$y = q^2 / (4\pi p^2)$$

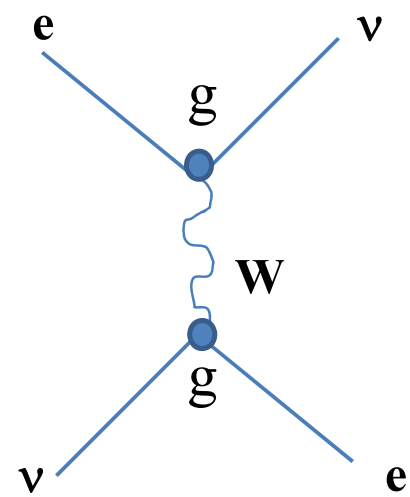
$$\frac{d\sigma}{dq^2}(e^- \mu^+ \to e^- \mu^+, y \to 0) = \frac{4\pi\alpha^2}{q^4}$$

$$t = q^2 = 4p^2 \sin^2(\theta/2)$$

- y→0
  - No muon recoil ⇒ Rutherford Scattering Formula

- Basic essence of lecture is all shown here.
  - You are ready to look into neutrino cross sections with this knowledge. If you cannot follow my lecture, please review
    - "Introduction to High Energy Physics" by D.H. Perkins, Chapter 5
    - "Quarks and Leptons" by F. Halzen and A.D. Martin, Chapter 6 and 12

- QED → Weak Interaction



$$M_{if} = \frac{\left(g/\sqrt{2}\right)^2}{q^2 + M_W^2} \approx \frac{g^2}{2M_W^2}$$

$$\frac{d\sigma}{dq^2}(\nu e^- \to \nu e^-) = \frac{2\pi}{v}\left|M_{if}\right|^2 p^2 \frac{dp}{dE_f}\frac{d\Omega}{dq^2}\frac{2}{(2\pi)^3}$$

$$= \frac{2}{\pi}\left(\frac{g^2}{8M_W^2}\right)^2 = \frac{G^2}{\pi}, \quad G \equiv \frac{\sqrt{2}g^2}{8M_W^2}$$

G=1.16639(1)×10^{-5}GeV^{-2}

$$\frac{d\sigma}{dy}\sigma(\nu e^- \to \nu e^-) = \frac{G^2}{\pi}s, \quad \sigma(\nu e^- \to \nu e^-) = \frac{G^2}{\pi}s$$

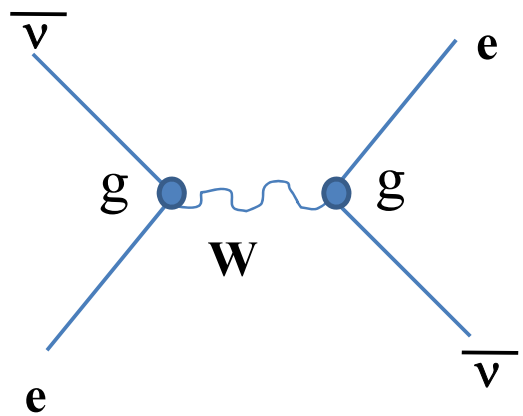
$$y = q^2/4p^2 = q^2/s$$

$$sdy = dq^2$$

$$s = (p_\nu + p_e)^2 = 2m_e E_\nu + m_e^2 \approx 2m_e E_\nu$$

The cross section is proportional to the neutrino energy!

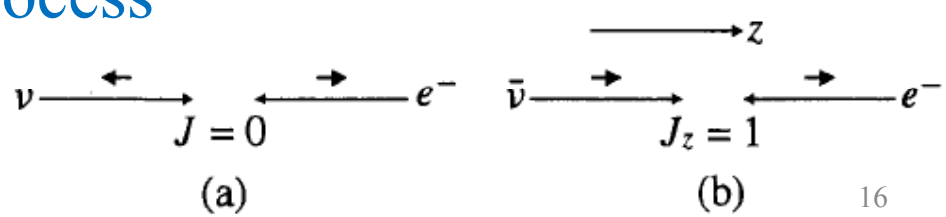
- How is anti-neutrino?
  - s→-t



$$\frac{d\sigma}{dy}\sigma(\bar{\nu}e^- \to \bar{\nu}e^-) = \frac{G^2 s}{\pi}(1-y)^2$$

$$\sigma(\bar{\nu}e^- \to \bar{\nu}e^-) = \frac{G^2}{3\pi}s = \frac{1}{3}\sigma(\nu e^- \to \nu e^-)$$

  - At y=(E_e/E_ν)=1, the cross section is zero.
  - The average energy is $E_\nu/4$. (Homework?)
  - Review the QED process



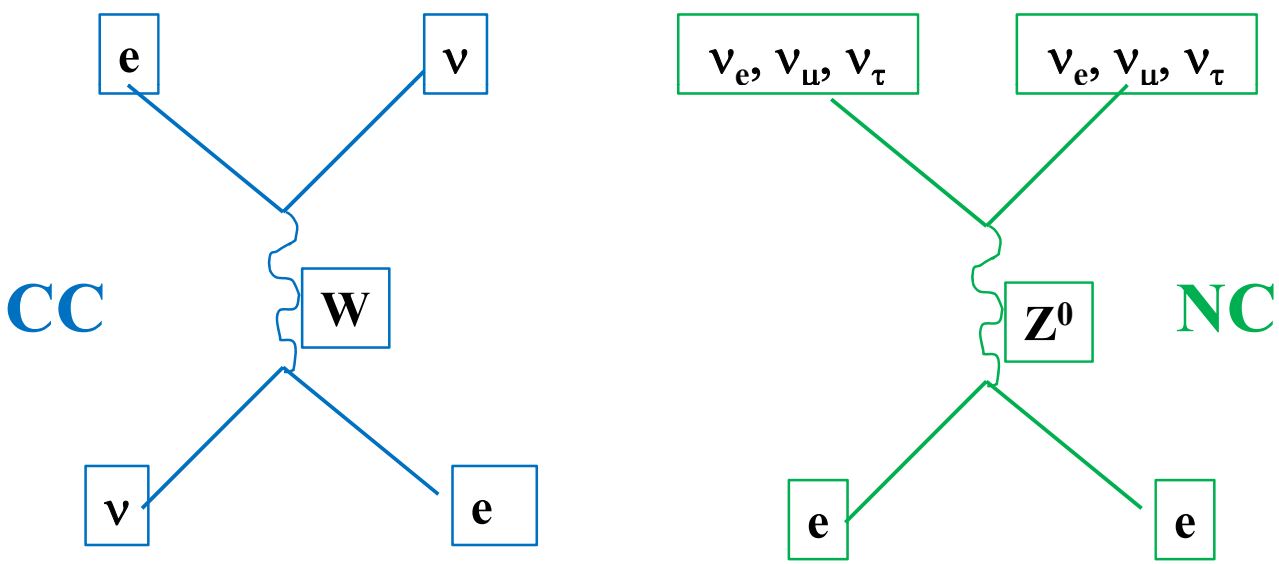
Charged+Neutral Current

$$H_W^{eff} = \frac{G_F}{\sqrt{2}}\left\{\left[\bar{e}\gamma_\mu(1-\gamma_5)\nu_e\right]\left[\bar{\nu}_e\gamma^\mu(1-\gamma^5)e\right] + \boxed{2\rho\left[\bar{\nu}_l\gamma_\mu\frac{1}{2}(1-\gamma_5)\nu_l\right]\left[\bar{e}\gamma^\mu(g_V - g_A\gamma_5)e\right]}\right\}$$

$$-i\frac{g}{\cos\theta_W}(J_\mu^{\ 3} - \sin^2\theta_W J_\mu^{\ EM})Z^\mu$$

$$g_V = g_L + g_R = T_3 - 2\sin^2\theta_W \cdot Q = -\frac{1}{2} + 2\sin^2\theta_W$$

$$g_A = g_L - g_R = T_3 = -\frac{1}{2} \qquad \sin^2\theta_W = 0.223$$



NC couplings

$$\sin^2\theta_W = 0.223$$

	g_L	g_R
e, μ, τ	$-\frac{1}{2}+\sin^2\theta_W$	$\sin^2\theta_W$
ν	½	0
u,c,t	$\frac{1}{2}-2/3*\sin^2\theta_W$	$-2/3*\sin^2\theta_W$
d,s,b	$-\frac{1}{2}+1/3*\sin^2\theta_W$	$1/3*\sin^2\theta_W$

$$-i\frac{g}{\cos\theta_W}(J_\mu^{\ 3}-\sin^2\theta_W J_\mu^{\ EM})Z^\mu$$

$$H_W^{eff} = \frac{G_F}{\sqrt{2}}\left\{\left[\bar{e}\gamma_\mu(1-\gamma_5)\nu_e\right]\left[\bar{\nu}_e\gamma^\mu(1-\gamma^5)e\right] + 2\rho\left[\bar{\nu}_l\gamma_\mu\frac{1}{2}(1-\gamma_5)\nu_l\right]\left[\bar{e}\gamma^\mu(g_V - g_A\gamma_5)e\right]\right\}$$

$$= \frac{G_F}{\sqrt{2}}\left[\bar{\nu}_l\gamma_\mu\frac{1}{2}(1-\gamma_5)\nu_l\right]\left[\bar{e}\gamma^\mu(c_V - c_A\gamma_5)e\right]$$

- $c_V = \rho g_V + 1$ $\nu_e e^-$ ($\nu_\mu e^-$: no -1: NC only)
- $c_A = \rho g_A + 1$ $\nu_e e^-$ ($\nu_\mu e^-$: no -1: NC only)
  - $c_L = ½*(c_V + c_A) = g_L + 1$ for ν_e
    $= g_L$ for ν_μ
  - $c_R = ½*(c_V - c_A) = g_R$

$$\sin^2\theta_W = 0.223$$

	c_L	c_R	$c_L^2 + \frac{1}{3}c_R^2$	$\frac{1}{2}c_L c_R$
$\nu_e e^-$	$1/2 + \sin^2\theta_W$	$\sin^2\theta_W$	0.5525	0.0845
$\bar{\nu}_e e^-$	$\sin^2\theta_W$	$1/2 + \sin^2\theta_W$	0.2317	0.0845
$\nu_\mu e^-$	$-1/2 + \sin^2\theta_W$	$\sin^2\theta_W$	0.0901	0.0311
$\bar{\nu}_\mu e^-$	$\sin^2\theta_W$	$-1/2 + \sin^2\theta_W$	0.0775	0.0311

$$\sigma = \frac{2G^2 m_e E_\nu}{\pi}\left[c_L^2 + \frac{1}{3}c_R^2 - \frac{1}{2}c_L c_R \frac{m_e}{E_\nu}\right]$$

$$\sigma(\nu_e e^- \to \nu_e e^-) = 0.952 \times 10^{-43}(E_\nu/10\,\mathrm{MeV})\ \mathrm{cm}^2$$
$$\sigma(\bar{\nu}_e e^- \to \bar{\nu}_e e^-) = 0.399 \times 10^{-43}(E_\nu/10\,\mathrm{MeV})\ \mathrm{cm}^2$$
$$\sigma(\nu_\mu e^- \to \nu_\mu e^-) = 0.155 \times 10^{-43}(E_\nu/10\,\mathrm{MeV})\ \mathrm{cm}^2$$
$$\sigma(\bar{\nu}_\mu e^- \to \bar{\nu}_\mu e^-) = 0.134 \times 10^{-43}(E_\nu/10\,\mathrm{MeV})\ \mathrm{cm}^2$$

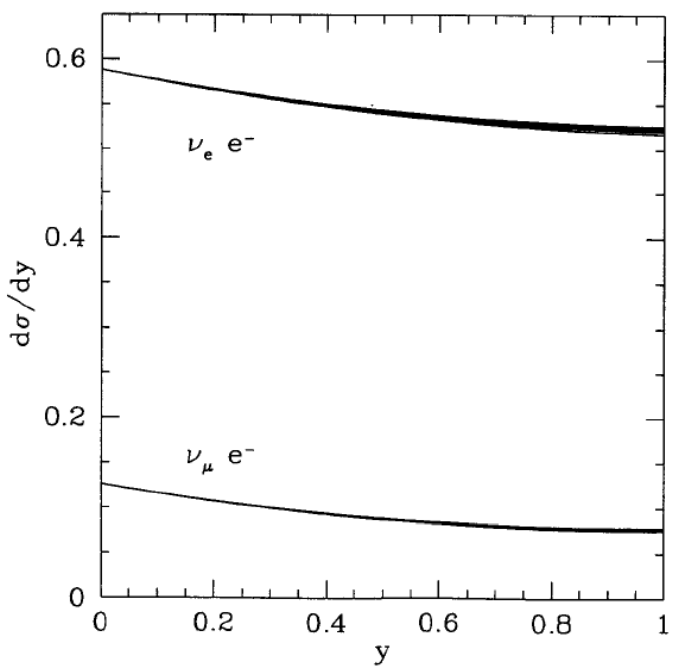


Fig. 3.5. Differential cross section of $\nu_e e^-$ and $\nu_\mu e^-$ scattering for $E = 5$–$10\,\mathrm{MeV}$. The curve is moving upwards (downwards) slightly with the energy of neutrinos for $\nu_e e$ ($\nu_\mu e$) scattering.

2. Cross Sections of ~MeV neutrinos

- Neutrino Sources
  - Solar Neutrinos (ν_e)
  - Reactor Neutrinos ($\overline{\nu_e}$)
  - Supernova Neutrinos (ν_e, $\overline{\nu_e}$, ν_μ, $\overline{\nu_\mu}$, ν_τ, $\overline{\nu_\tau}$)

- How to detect the neutrinos?

Solar Neutrinos

- Water Cherenkov Detector (Super-K, …)
  - $\nu + e \rightarrow \nu + e$ scattering, and observe the electron.
  - You already learn
    - $\sigma(\nu_e + e \rightarrow \nu_e + e) \sim 6\times \sigma(\nu_\mu + e \rightarrow \nu_\mu + e)$
  - How many neutrino events / day [Homework?]
  - Why do experimentalists work hard to lower the threshold?
    - Super-K: E_{th}~5MeV or less
    - Kamiokande: E_{th}~6.5MeV
    - Super-K-II E_{th}~7.0MeV

Table 3.6. Neutrino electron scattering for ^{8}B solar neutrinos. Detection fractions are shown for the given threshold energy for recoil electrons. Radiative corrections are not taken into account.

T'_{th} (MeV)	$\nu_e e$	$\nu_\mu e$	$\sigma(\nu_e e)/\sigma(\nu_\mu e)$
4	42.9%	38.2%	6.64
4.5	37.0%	32.7%	6.69
5	31.6%	27.8%	6.72
5.5	26.7%	23.3%	6.77
6	22.2%	19.2%	6.83
6.5	18.2%	15.6%	6.89
7	14.6%	12.5%	6.90
7.5	11.5%	9.8%	6.93
8	8.9%	7.6%	6.92
10	2.4%	2.0%	7.02
$\sigma_{\rm tot}(\rm cm^2)$	6.39×10^{-44}	1.08×10^{-44}	5.91

- Detection of anti-neutrinos.
  - Is the cross section smaller than $\nu+e\rightarrow\nu+e$ scattering?
    - $\sigma(\nu_e+e\rightarrow \nu_e+e) \sim 3\times \sigma(\bar{\nu}_e+e\rightarrow \bar{\nu}_e+e)$
  - We can use a proton in H_2O or CH(KamLAND, ..)!
    - Inverse beta decay: $\bar{\nu}p\rightarrow e^+n$.
    - Native expectation (assuming a proton as a Dirac particle):

$$\sigma(\bar{\nu}e^- \rightarrow \bar{\nu}e^-) = \frac{G^2}{3\pi}s \Rightarrow \sigma(\bar{\nu}p \rightarrow e^+n) \sim \frac{G^2}{3\pi}s$$

$$s = (p_\nu + p_{e(p)})^2 = 2m_{e(p)}E_\nu + m_{e(p)}^2 \approx 2m_{e(p)}E_\nu$$

    - In reality, the proton is NOT a Dirac particle.

$$H_W^{eff} = \frac{G_F \cos\theta_C}{\sqrt{2}}\left[\bar{e}\gamma_\mu(1-\gamma_5)\nu_e\right]\left[\bar{u}(p')\gamma^\mu(1-g_A\gamma^5)u(p)\right]$$

- The matrix element (same as β decay) is

$$\frac{1}{2}|T|^2 = \frac{G_F^2 \cos^2\theta_C}{2} 8E_e E_\nu \left[(1+\beta\cos\theta) + 3g_A^2 \left(1 - \frac{\beta}{3}\cos\theta\right)\right]$$

$$\sigma(\bar{\nu}_e p \to e^+ n) = \frac{1}{2E_\nu}\int \frac{d^3 p_e}{2E_e (2\pi)^3}(2\pi)\delta(E_\nu - E_e + m_p - m_n)\frac{1}{2}|T|^2$$

$$= \frac{G_F^2 E_e E_p}{\pi}\cos^2\theta_C (1 + 3g_A^2)$$

$$\approx 9.30 \times 10^{-42}\left(\frac{E_\nu}{10\ MeV}\right)^2 cm^2$$

No free neutron

– NOTE: $\sigma(\bar{\nu}_e p \to e^+ n) = \sigma(\nu_e n \to e^- p)$

$\cos\theta_C = 0.974$, $g_A = 1.267$

$d\sigma/d\theta \propto 1 - 0.104\cos\theta$

Well known process and reliable calculation for the low energy anti-neutrino (~MeV)!

- For the higher energy anti-neutrinos, we must take into account of the nucleon structure (form factor).
- **Form Factor of the proton:**
  - Fourier transformation of the charge distribution.
  - In the case of exponential distribution:

    ρ (r)=ρ(0)exp(-mr)

$$F\left(|q|^2\right) = N\int e^{-mr} e^{i\vec{q}\cdot\vec{x}} d^3x = N\int r^2 dr e^{-mr}(2\pi)\int_{-1}^{1} d(\cos\theta) e^{i|q|r\cos\theta}$$

$$= \frac{2\pi N}{i|q|}\int_0^\infty r dr\left[e^{-(m-i|q|)r} - e^{-(m+i|q|)r}\right] = \frac{8\pi N}{m^3\left(1+|q|^2/m^2\right)^2}$$

Normalization: $N\int e^{-mr} d^3x = 1 \Rightarrow N = m^3/8\pi$

$$F\left(q^2\right) = \frac{1}{\left(1-\frac{q^2}{m^2}\right)^2}$$

- In QEC (ep scattering), we can study vector form factor.
- In the neutrino scattering, we study both vector and axial-vector form factors.
  - M_V=0.84GeV/c^2
    - The rms radius of the charge distribution is **0.8 fm.**
  - M_A=1.05±0.05GeV/c^2

•$\overline{\nu}p \rightarrow e^+n$ is a good mode to detect the anti-neutrinos (Reactor and Super-nova) because the higher cross section and the association of a neutron

$$\sigma\left(\overline{\nu}_e p \rightarrow e^+ n\right) \approx 9.30 \times 10^{-42}\, cm^2$$

$$\sigma\left(\nu_e e \rightarrow \nu_e e\right) \approx 0.09 \times 10^{-42}\, cm^2$$

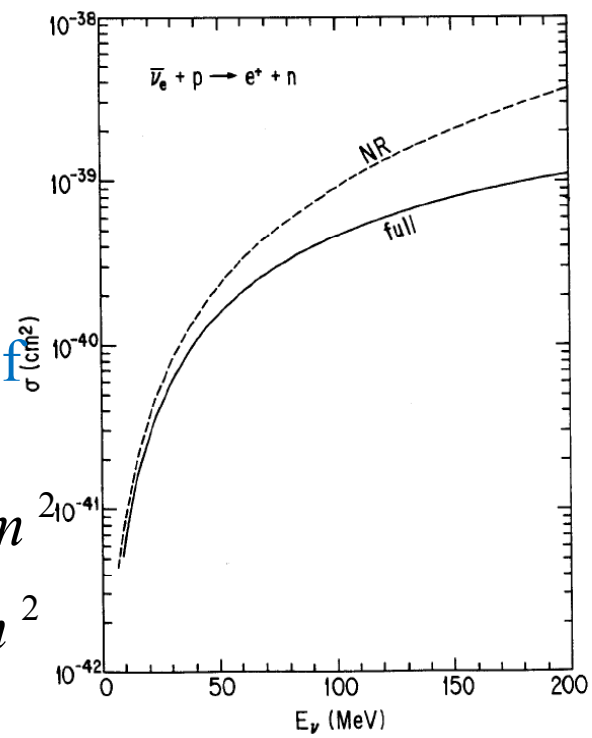


Fig. 3.6. Cross section of $\bar{\nu}_e + p \rightarrow e^+ + n$ calculated from the full expression (3.162). The nonrelativistic expression (NR) is also plotted for comparison.

- How to detect the low energy solar neutrinos?
  - Use a Nucleus

$$\sigma = \frac{G_F^2 \cos^2 \theta_C}{\pi} \sum_i \left| M_{\nu i} \right|^2 E_e \left(E_e^2 - m_e^2 \right)^{1/2} F\left(Z+1, E_e \right)$$

$M_{\nu i}$: Nuclear Matrix element.

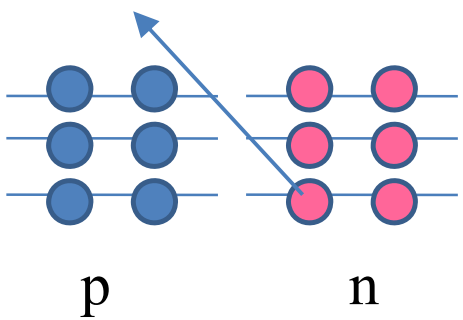
F : Fermi function for the correction of Coulomb attraction between the electron and nucleus.

The point is that the proton in $\nu + n \rightarrow e + p$ must be at some state in the nucleus.

  - We have to know the probability of transition between states.

- Example: Devis's experiment
  - $\nu_e + {}^{37}Cl \rightarrow e^- + {}^{37}Ar$
  - The lowest ${}^{37}Ar$ state is not high transition probability.
    - Naive idea: The lowest state is often occupied by a proton in advance.
    - The coupling must be known.

$\nu + n \rightarrow e + p$



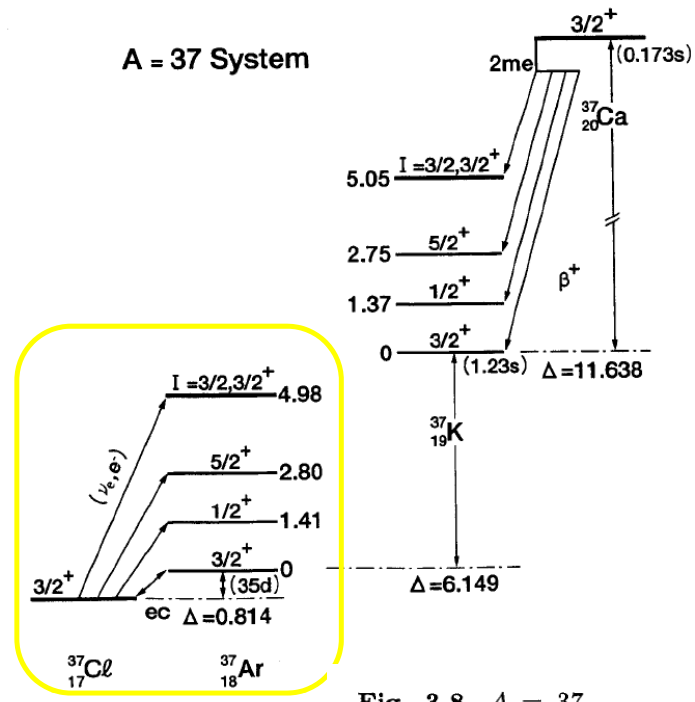


Fig. 3.8. $A = 37$ ${}^{37}_{17}Cl - {}^{37}_{18}A - {}^{37}_{19}K - {}^{37}_{20}Ca$ system. Energy levels and the Q value Δ are in units of MeV. Levels denoted by thick lines are the $I = 3/2$ isobaric analogue states.

Table 3.7. Neutrino (ν_e) captures in nuclei. The threshold energy for E_ν and cross section (for $E_\nu \sim 35$ MeV) are shown.

		n	^{2}H	^{12}C	^{16}O	^{37}Cl	^{56}Fe
$E_{\rm th}$	(MeV)	−0.78	1.44	17.34	15.43	0.814	4.57
σ	($\times 10^{-41}$cm^2)	11.4	5.9	1.50[a]	0.64	8.3	28
σ/neutron	($\times 10^{-42}$cm^2)	114	59	2.5	0.80	4.2	9.3
Ref.		—	[413]	[414]	[415]	[416]	[417]

[a] With the aid of experimental information.

- The cross section is not the sum of neutrons.
- The accurate estimation of the cross section is very difficult.
  - Need the experimental input of the nuclear matrix element.

Table 3.9. Comparison of the Gamow–Teller matrix elements $g_A^2 \langle \sigma \rangle^2$ for $^{37}\mathrm{Cl}(\nu_e, e^-)^{37}\mathrm{A}$.

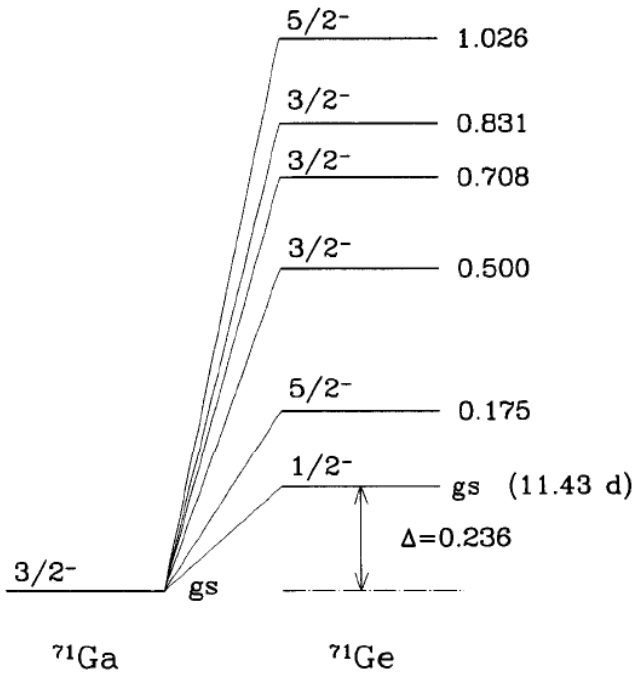
^{37}Cl level (keV)	^{37}K level (keV)	J^P	$^{37}\mathrm{Ca} \to {}^{37}K\beta^+$ Sextro et al. [420]	$^{37}\mathrm{Ca} \to {}^{37}\mathrm{K}\beta$ García et al. [421]	$^{37}\mathrm{Ca} \to {}^{37}\mathrm{K}\beta$ Trinder et al. [434]	*sd* shell[a] [369]
0	0	$3/2^+$	0.0493	0.0483	0.048(2)	0.072
1410	1371	$1/2^+$	0.0510	0.074(10)	0.013(1)	0.034
2796	2750	$5/2^+$	0.0833	0.067(4)	0.102(3)	0.273
3170	3239	$(5/2)^{+b}$	—	0.004(1)	0.087(4)	0.058
3602	3622	$3/2^+$	0.0907	0.075	0.073(2)	0.157
?	3840	?	0.118	0.094(5)	0.093(3)	$\times^c$
?	4191	?	—	0.002(1)	—	$\times^c$
?	4413	?	0.0464	0.043	—	$\times^c$
?	4495	$1/2^+$	0.0894	0.060	—	0.356
?	4605	?	0.0561	—	—	$\times^c$
?	5016	?	0.1186	0.064(64)	—	$\times^c$
4980	5051	$3/2^+$	3.083*	3.10(10)*	—	3.119*
?	5120	$1/2^+$	0.414	0.47(2)	—	$\times^c$
5130	5320	$3/2^+$	0.0868	0.048(3)	—	0.131

heigt4mm*The Fermi transition contribution is included.
[a] $g_A^{\mathrm{eff}} = 1$ is used.
[b] Spin-parity assignment according to the shell model.
[c] Levels that do not appear in the *sd* shell model.

Dominant transition

- The cross section estimate is not simple.
- The differential cross section as a function of energy is not simple, neither.

- In order to measure the lower energy neutrinos:
  - ^{71}Ge is the ideal target nucleus.



	$pp\nu$	other ν's	Sum
Ground state	69.7	43.1	112.8
Excited states	0.0	15.1	15.1
Sum	69.7	58.2	128

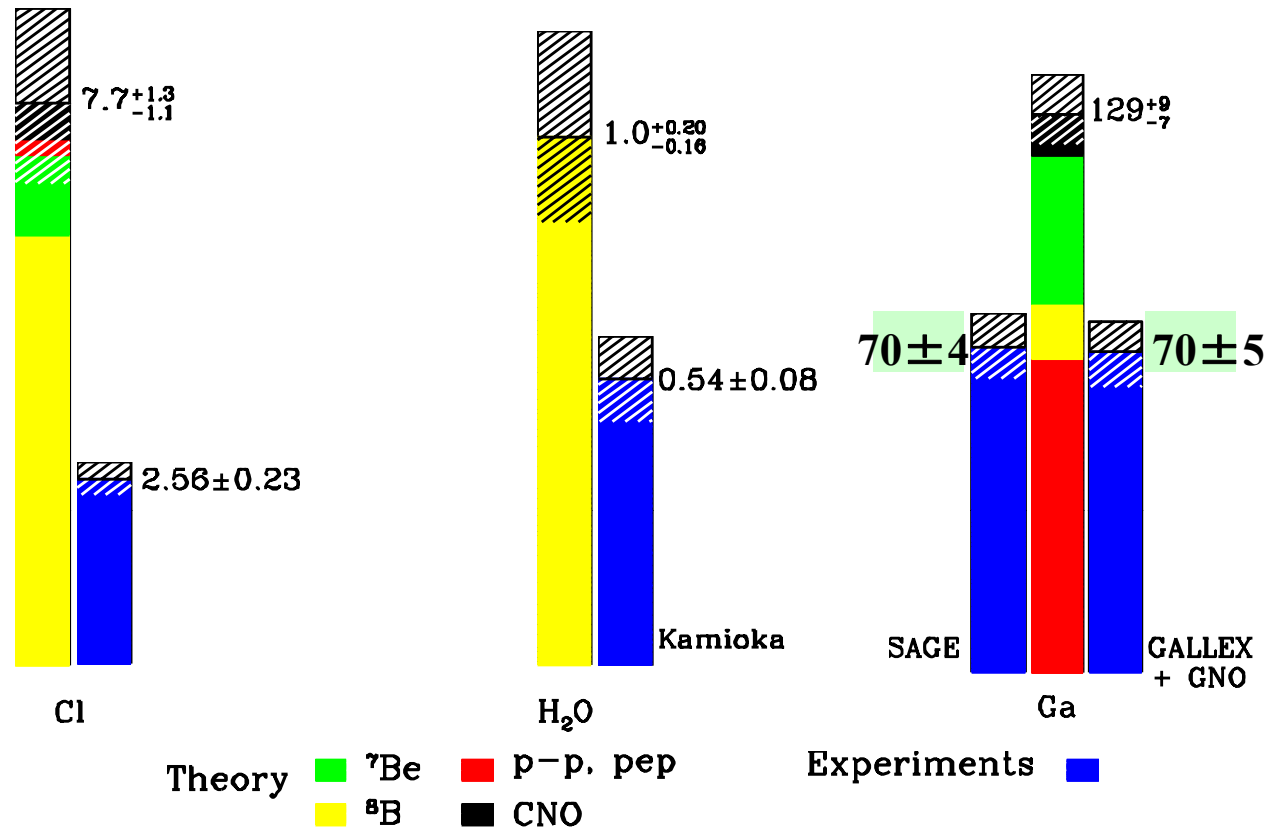
  - Gallex(GNO), SAGE

Table 3.11. Cross sections for $^{37}\mathrm{Cl}(\nu_e, e^-)\,^{37}\mathrm{A}$ and for $^{71}\mathrm{Ga}(\nu_e, e^-)^{71}\mathrm{Ge}$ at low energies (in units of $10^{-42}\mathrm{cm}^2$, taken from [398, 445].

E_ν	$\sigma(^{37}\mathrm{Cl})$	E_ν	$\sigma(^{71}\mathrm{Ge})$	E_ν	$\sigma(^{71}\mathrm{Ge})$
1	5.21×10^{-4}	0.24	1.31×10^{-3}	4	0.213
2	3.70×10^{-3}	0.25	1.36×10^{-3}	5	0.438
3	1.02×10^{-2}	0.275	1.50×10^{-3}	6	0.843
4	2.23×10^{-2}	0.3	1.66×10^{-3}	7	1.53
5	5.38×10^{-2}	0.35	2.02×10^{-3}	8	2.58
6	0.144	0.4	2.41×10^{-3}	9	3.97
7	0.462	0.45	2.86×10^{-3}	10	5.71
8	1.01	0.5	3.31×10^{-3}	11	7.78
9	1.85	0.6	4.30×10^{-3}	12	10.2
10	3.00	0.7	5.40×10^{-3}	13	12.9
11	4.45	0.8	6.85×10^{-3}	14	15.9
12	6.21	0.9	8.28×10^{-3}	15	19.1
13	8.27	1.0	9.83×10^{-3}	16	22.6
14	10.6	1.2	1.44×10^{-2}	18	30.4
15	13.3	1.4	1.92×10^{-2}	20	39.0
16	16.2	1.6	2.55×10^{-2}	30	87.9
18	22.8	1.75	3.06×10^{-2}		
20	30.5	2.0	3.97×10^{-2}		
30	82.0	3.0	9.91×10^{-2}		

Total Rates: Standard Model vs. Experiment

Bahcall–Pinsonneault 2000



- Measurement of neutral current for neutrino oscillation.
  - $\nu_e \rightarrow \nu_e$ or ν_μ, ν_τ
  - Neutrino oscillation can be well confirmed by measuring ν_μ and ν_τ by NC.
- Deuterium (D_2O): SNO
  - CC: $\nu + d \rightarrow e^- + p + p$
  - NC: $\nu + d \rightarrow \nu + p + n$ (and detect the neutron)

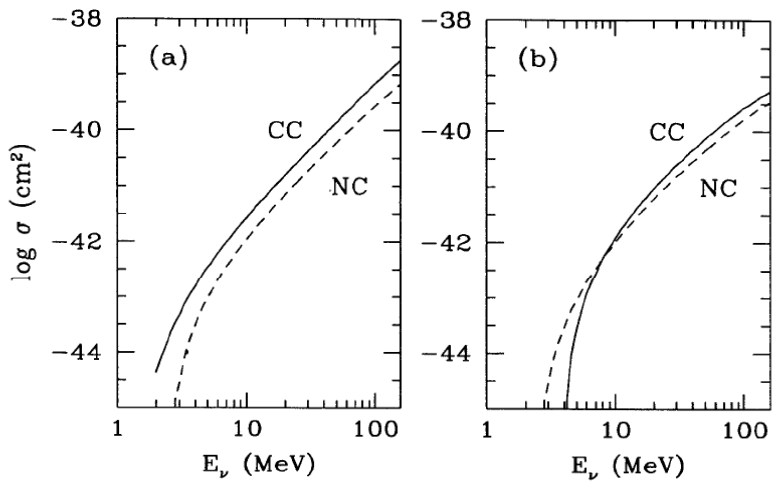
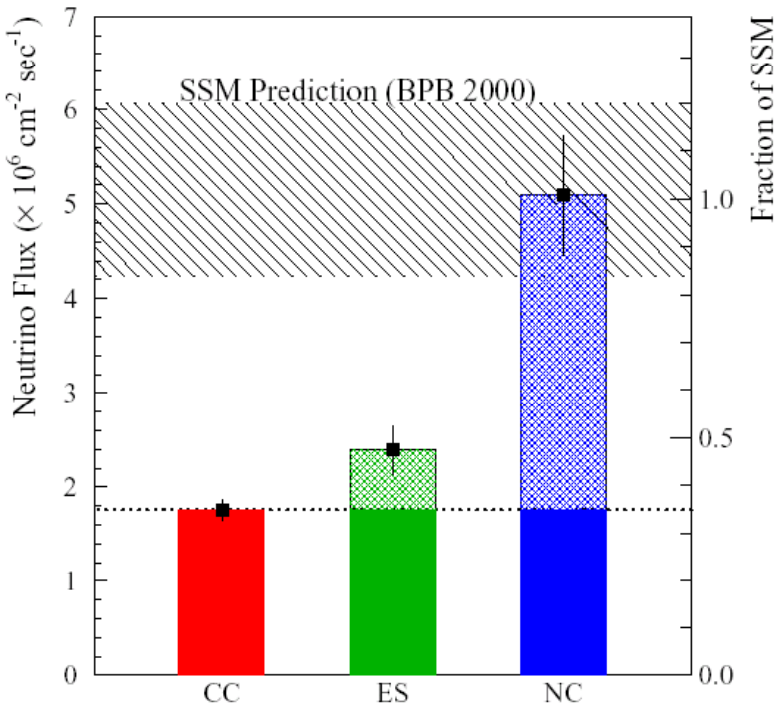


Fig. 3.13. Neutrino reaction cross sections on deuterium (**a**) $\nu_e + d \rightarrow e^- + p + p$ (*solid curve*) and $\nu + d \rightarrow \nu + p + n$ (*dashed curve*); (**b**) $\bar{\nu}_e + d \rightarrow e^+ + n + n$ (*solid curve*) and $\bar{\nu} + d \rightarrow \bar{\nu} + p + n$ (*dashed curve*). Data are taken from calculations of [413].

Measured SNO Fluxes



Assuming ^{8}B energy spectrum ...

Fluxes ($\times 10^6$ cm^{-2} sec^{-1})

$$\phi_{CC} = 1.76^{+0.06}_{-0.05}\ (\text{stat.}) \pm 0.09\ (\text{sys.})$$

$$\phi_{ES} = 2.39^{+0.24}_{-0.23}\ (\text{stat.}) \pm 0.12\ (\text{sys.})$$

$$\phi_{NC} = 5.09^{+0.44}_{-0.43}\ (\text{stat.})^{+0.46}_{-0.43}\ (\text{sys.})$$

$$\phi_{CC} < \phi_{ES} < \phi_{NC}$$

NC flux in agreement with SSM prediction!

Cross Sections of 10~100MeV neutrinos

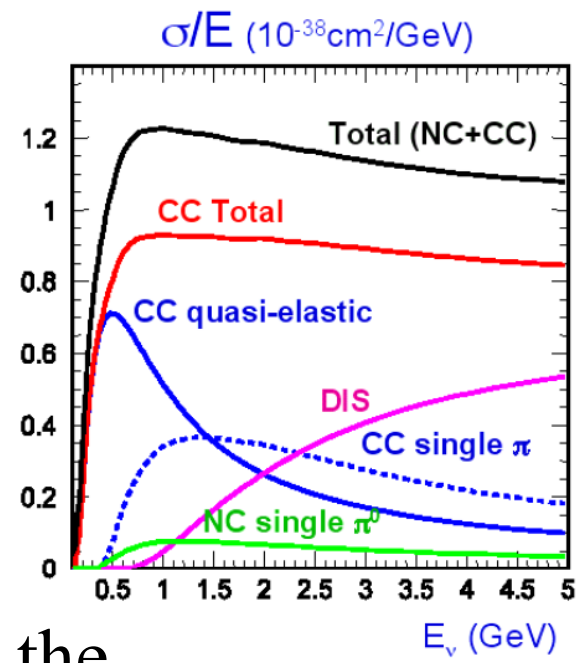
- Nucleus Target
  - Reliable calculation of cross section is not easy!

Table 3.17. Cross section of $\nu +$ ^{12}C reactions (in units of 10^{-42}cm^2) (continued).

Authors	Method	$\langle^{12}\mathrm{C}(\nu_\mu,\nu_\mu)^{12}\mathrm{C}\rangle$ stopped π	$\langle^{12}\mathrm{C}(\nu_\mu,\mu^-)^{12}\mathrm{N}\rangle$ π decay in flight
Donnelly-Peccei [453]	Shell model	2.7	
Fukugita et al. [414]	EPT	2.7	
Minz/Pourkaviani [455, 462]	EPT	2.7	
Engel et al. [463]	EPT		62.9
Kolbe et al. [465]	RPA	2.8	64–75
Engel et al. [463]	Shell model		63.5
Auerbach et al. [466]	RPA		39–50
Hayes and Towner [467]	Shell model		70–240
KARMEN [469]	Expt.	$3.2 \pm 0.5 \pm 0.4$	
LSND [470]	Expt.		$66 \pm 10 \pm 10$

3. Cross Sections of ~GeV neutrinos

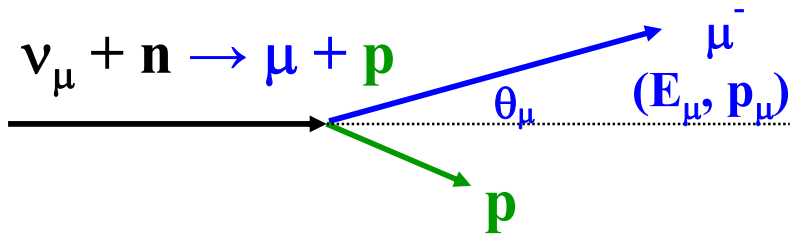
- It is the energy of accelerator neutrinos for the long baseline.
- Dominant Interaction Mode
  - Elastic scattering.
    - CC: Quasi-elastic scattering.
    - NC: Elastic scattering.
  - 1 π production with hadron resonance.
  - 1 π production in coherent scattering.
  - Deep inelastic scattering.



- Not only the baseline interaction, but the secondary interaction in the nucleus makes the story difficult
  - Tutorial by J. Morfin?

K2K Neutrino Energy E_ν Reconstruction

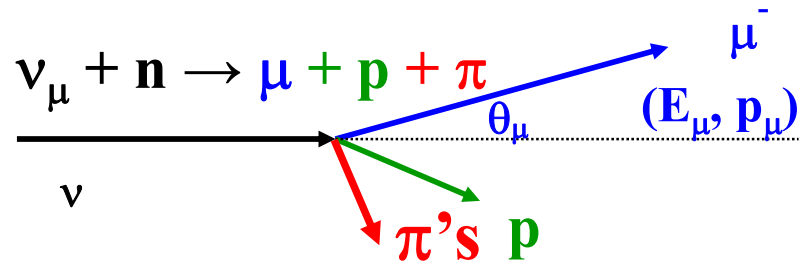
CC quasi elastic (QE)

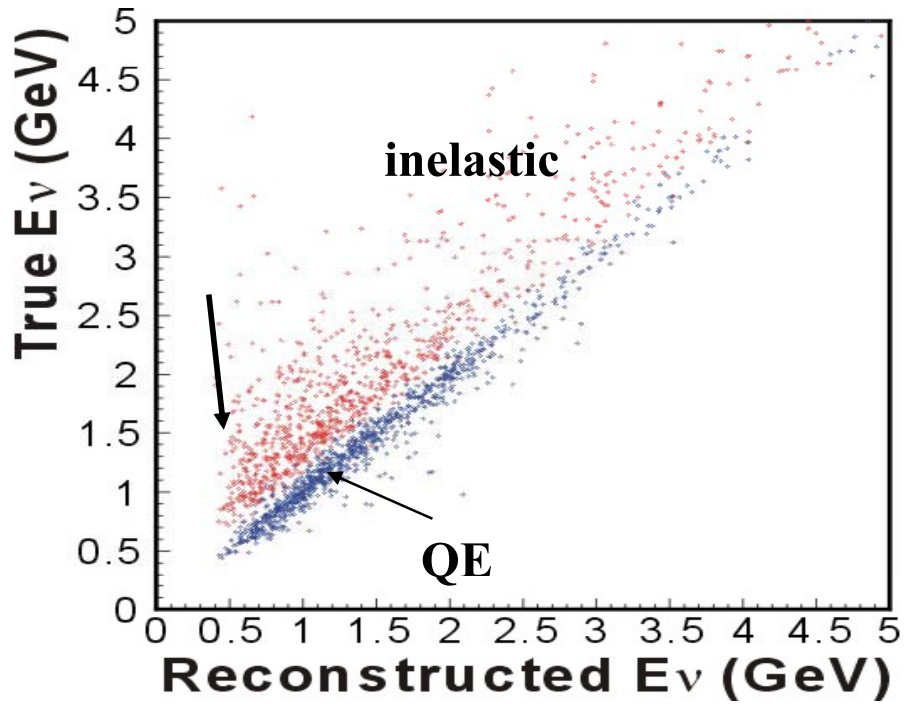


$$E_\nu = \frac{m_N E_\mu - m_\mu^2/2}{m_N - E_\mu + p_\mu \cos\theta_\mu}$$

$$\text{Rate}(E\nu,\text{Near}) \rightarrow \underline{\phi(E\nu,\text{Near})}$$
$$\uparrow$$
$$\sigma(\text{QE}),\ \sigma(\text{nonQE})$$

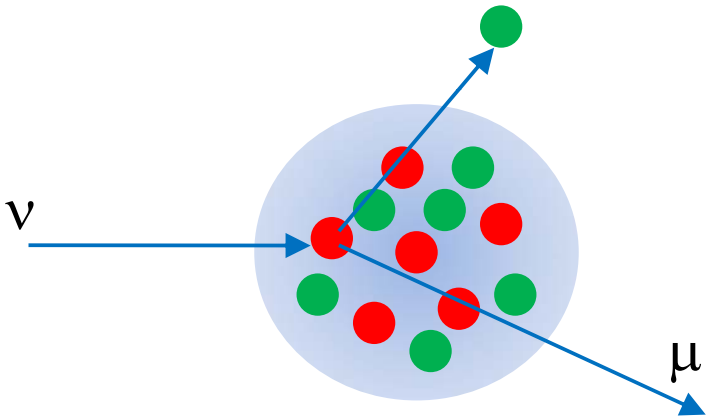
CC inelastic

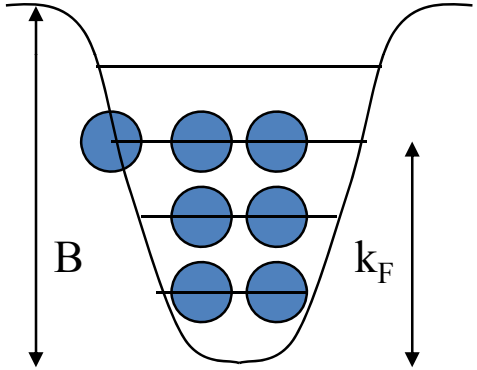




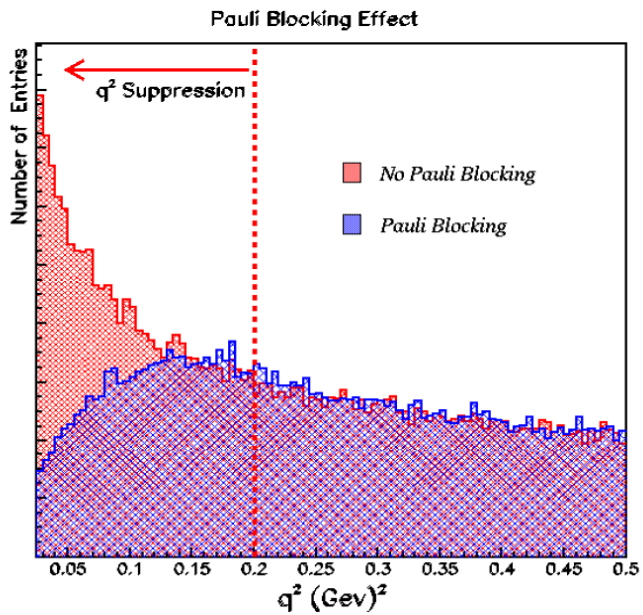
CC Quasi-elastic scattering with nucleus

- A Fermi gas model
  - The interaction take place through quasi-free scattering of nucleons contained in the non-interacting Fermil gas.





- Nucleons in the initial state have momentum (Fermi momentum) ~200MeV/c
- The nucleon in the final state must have the momentum above Fermi momentum surface.
  - Pauli blocking ⇒ low Q2 suppression.



- We must take into account of the form factors of a nucleon.

$$F_V\left(Q^2 = -q^2\right) = \frac{1}{\left(1 + \frac{Q^2}{M_V^2}\right)^2}$$

$$F_A\left(Q^2\right) = \frac{1}{\left(1 + \frac{Q^2}{M_A^2}\right)^2}$$

$$\frac{d\sigma}{dQ^2} = \frac{G^2 \cos^2\theta_C}{4\pi}\{(F_V + F_W + F_A)^2 + (F_V + F_W + F_A)^2\left(1 - \frac{Q^2}{2E_\nu m_N}\right)^2$$

$$+ \left[F_A^2 - (F_V + F_W)^2\right]\frac{Q^2}{2E_\nu^2}$$

$$+ \left[F_W^2 \frac{(Q^2 + 4m_N^2)}{4m_N^2} - 2(F_V + F_W)F_W\right]\left[2 - \frac{Q^2(m_N + 2E_\nu)}{2E_\nu^2 m_N}\right]\}$$

$$F_W(Q^2) \propto F_V(Q^2)$$

F_W:Weak magnetism tensor term

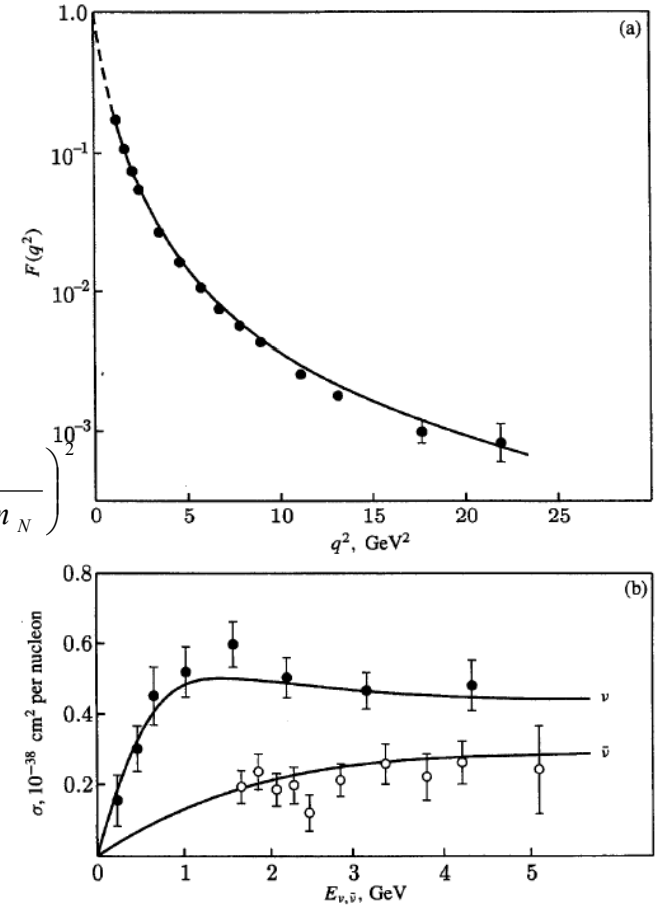
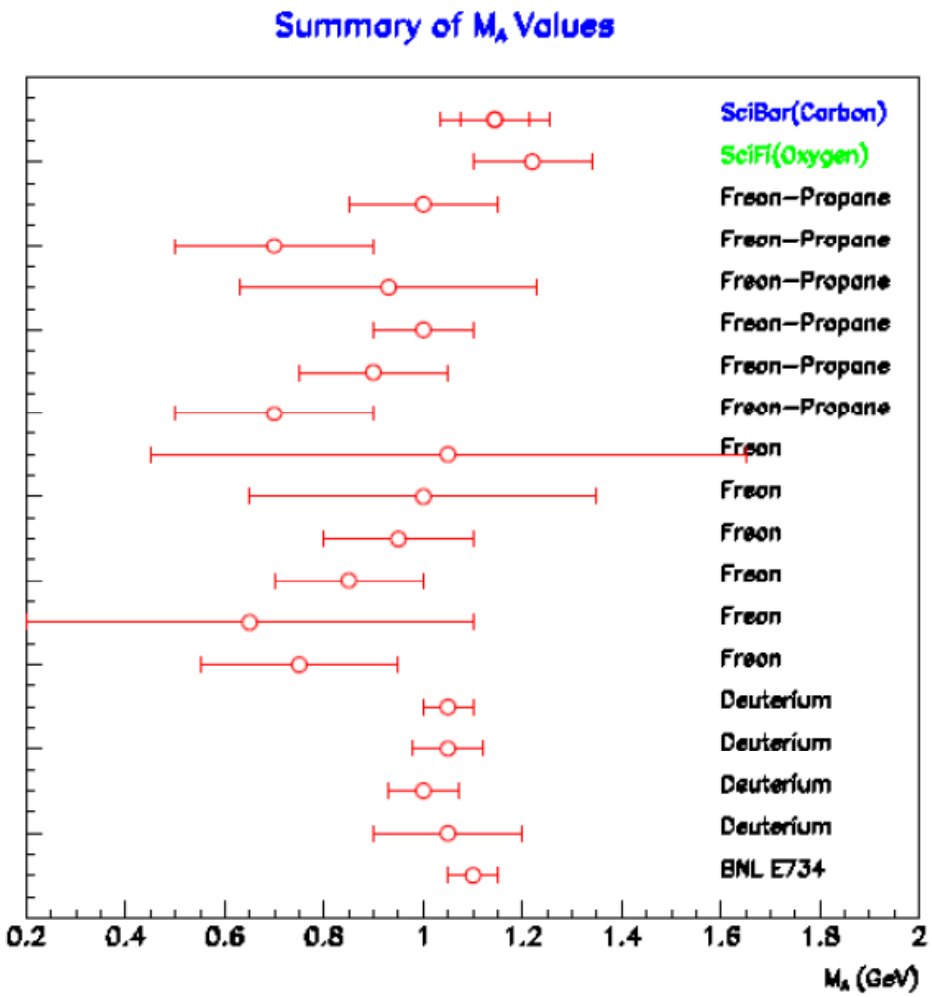
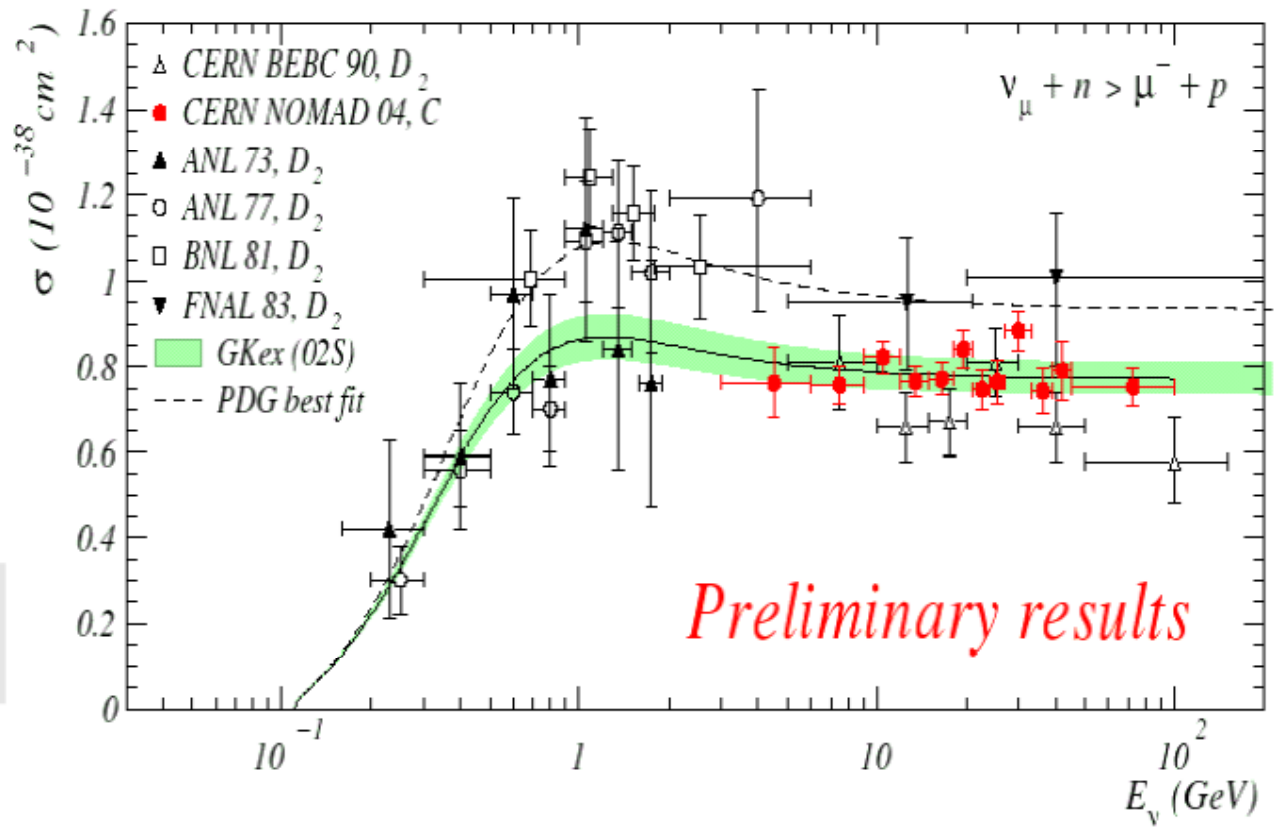


Fig. 5.9. (a) The form factor of the proton as measured in electron–proton scattering. At high q^2, this is dominated by the distribution of magnetic moment, rather than charge, on the proton. The curve represents the dipole formula (5.28) with $M_V = 0.9$ GeV. (b) The cross-section for the quasi-elastic reactions $\nu_\mu + n \to \mu^- + p$ (solid circles) and $\bar{\nu}_\mu + p \to \mu^+ + n$ (open circles). The data comes from measurements in bubble chambers at CERN with freon fillings and from the Argonne National Laboratory (ANL) with deuterium fillings. The curves are for $M_A = M_V = 0.9$ GeV.



- I wrote that M_A=1.05±0.05 GeV.
- However, each measurement has large error, and there may be nuclear dependence.

- CC quasi-elastic is one of the most well measured cross section in this energy range. Even so, the precision is NOT great!
  - One of the study items in the next generation experiment.



- CC 1π production: νN→μNπ.
  - The amplitude is sum of I=1/2 and 3/2 processes contributions.

$$A(\nu N \rightarrow \mu N \pi) = \sum_{I_s = 1/2, 3/2} c_{I_s} A_{I_s}$$

$$A(\nu N \rightarrow \mu^- \Delta^{++} \rightarrow \mu^- p \pi^+) = A_{3/2}$$

$$A(\nu N \rightarrow \mu^- p \pi^0) = \sqrt{2}/3\ A_{3/2} - \sqrt{2}/3\ A_{1/2}$$

$$A(\nu N \rightarrow \mu^- n \pi^+) = 1/3\ A_{3/2} + 2/3\ A_{1/2}$$

  - $C_{3/2}$ for Δ

$$\left|\frac{3}{2},\frac{3}{2}\right\rangle = \pi^+ p$$

$$\left|\frac{3}{2},\frac{1}{2}\right\rangle = \sqrt{\frac{1}{3}}\pi^+ n + \sqrt{\frac{2}{3}}\pi^0 p$$

$$\left|\frac{3}{2},-\frac{1}{2}\right\rangle = \sqrt{\frac{2}{3}}\pi^0 n + \sqrt{\frac{1}{3}}\pi^- p$$

$$\left|\frac{3}{2},-\frac{3}{2}\right\rangle = \pi^- n$$

- The scattering matrix element through Δ^{++} is

$$T = \frac{G_F}{2}\langle \Delta \mid V_\lambda - A_\lambda \mid N \rangle \bar{\nu}_\mu(k')\gamma^\lambda(1-\gamma_5)\mu(k)$$

- By introducing form factors,

$$\frac{d\sigma}{dt} \approx \frac{G^2}{24\pi}\frac{s - m_\Delta^2}{s - m_N^2}\left(\frac{m_\Delta + m_N}{m_\Delta}\right)^2 \left|C_5^A(t)\right|^2$$

$$\sigma = \frac{G^2}{72\pi}\left|C_5^A(t)\right|^2\left(\frac{m_A}{m_N}\right)^2\left[(m_\Delta + m_N)^2 + m_A^2/2\right]$$

- $\sigma \sim 4\times10^{-38}\mathrm{cm}^2$ is good agreement with the $\nu p \rightarrow \mu^- p \pi^+$ experiment for $m_{p\pi} < 1.4\mathrm{GeV}$.

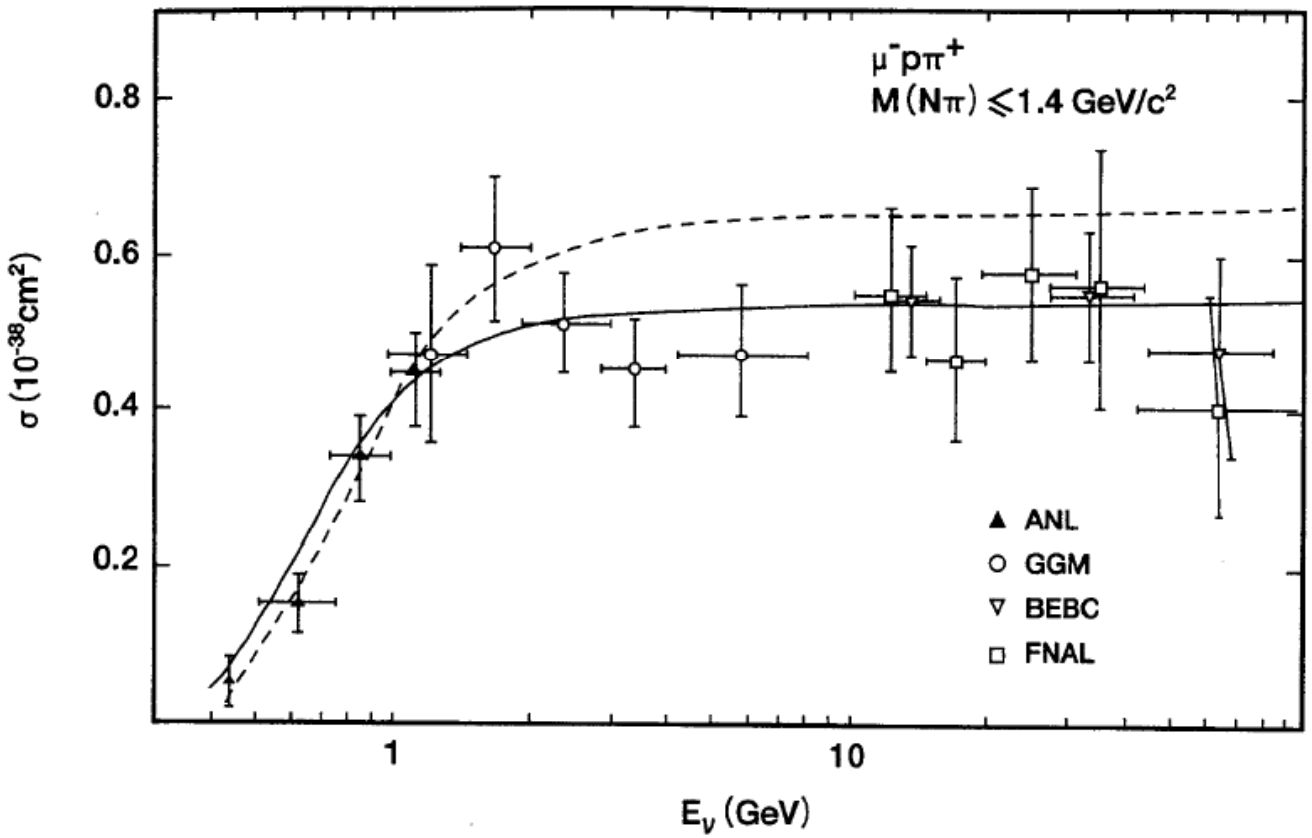


Fig. 3.16. Cross section of $\nu p \to \mu^- p\pi^+$ as a function of the energy of the neutrino for the final hadron invariant mass of $M(N\pi) \leq 1.4$ GeV. The *solid curve* is the prediction of Fogli and Narduli [501], and the *dashed* is that of Rhein and Sehgal [508], compared with data taken from BEBC [502], FNAL [503], ANL [504], and Gargamelle (GGM) [505].

- For Δ resonance, we have 14 final state overall (6CC and 8NC)

CC

$\nu_\mu p \to \mu^- \Delta^{++}$	$\Delta^{++} \to p\pi^+$
$\nu_\mu n \to \mu^- \Delta^+$	$\Delta^+ \to p\pi^0, \Delta^+ \to n\pi^+$
$\overline{\nu}_\mu p \to \mu^+ \Delta^0$	$\Delta^0 \to n\pi^0, \Delta^0 \to p\pi^-$
$\overline{\nu}_\mu n \to \mu^+ \Delta^-$	$\Delta^- \to n\pi^-$

NC

$\nu_\mu p \to \nu_\mu \Delta^+$	$\Delta^+ \to p\pi^0, \Delta^+ \to n\pi^+$
$\nu_\mu n \to \nu_\mu \Delta^0$	$\Delta^0 \to n\pi^0, \Delta^0 \to p\pi^-$
$\overline{\nu}_\mu p \to \overline{\nu}_\mu \Delta^+$	$\Delta^+ \to p\pi^0, \Delta^0 \to n\pi^+$
$\overline{\nu}_\mu n \to \overline{\nu}_\mu \Delta^0$	$\Delta^0 \to n\pi^0, \Delta^0 \to p\pi^-$

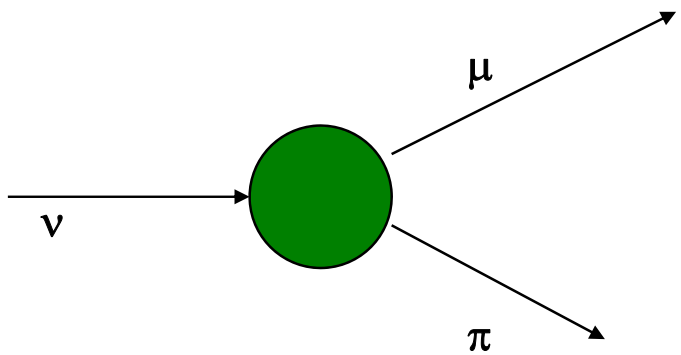
- For other π production process, we must consider other Δ resonance including off-shell Δ's.
  - What is the effect of $A_{1/2}$?
  - $R=|A_{1/2}|/|A_{3/2}|=0.68\pm0.04$
    - The experimental result is consistent with a resonant I=3/2 amplitude in the presence of a large non-resonant I=1/2 background.
    - Experimental results:

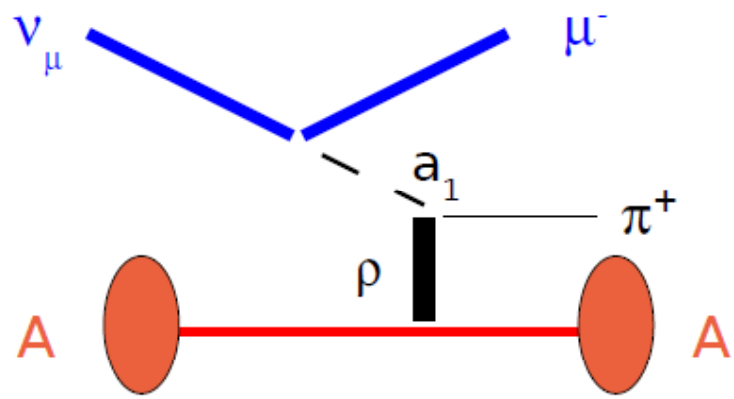
$$\frac{\sigma(\mu^- n\pi^+)}{\sigma(\mu^- p\pi^0)} = \frac{1}{2} \quad \text{for } A_{3/2}$$

but experimental result is 0.96±0.12

Coherent π production

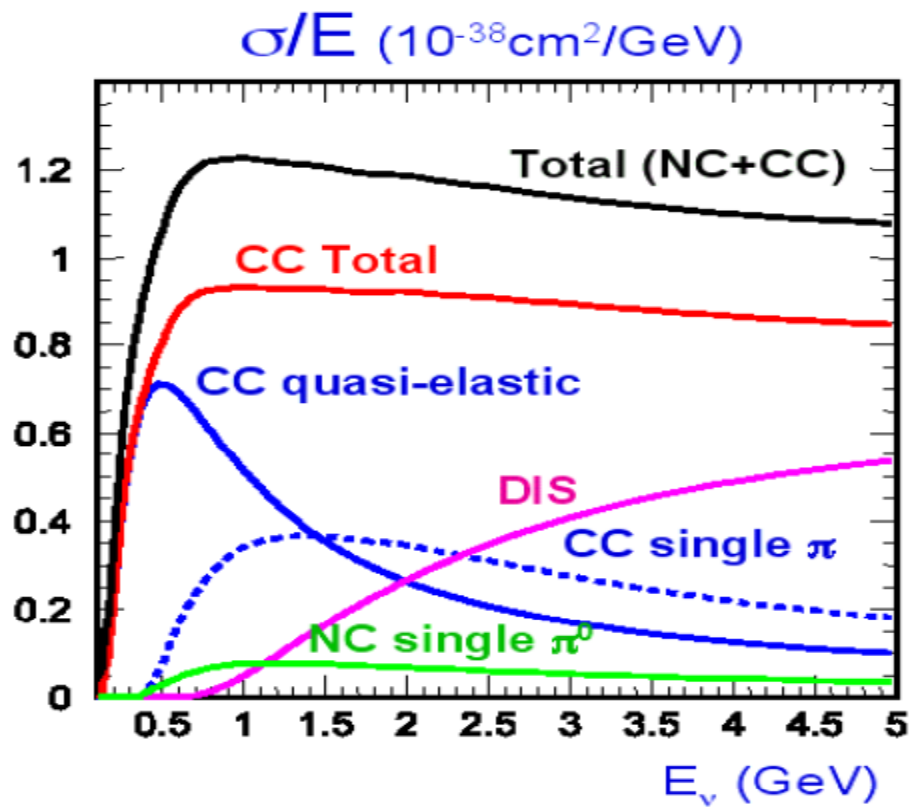
CC-coherent π (ν+A→μ+A+π)





Neutrino interacts coherently with nucleons bound in the nucleus, producing pion.

- may be too details as a lecture.



- DIS (Deep Inelastic Scattering) will be discussed in the next chapter since it is a dominant cross section at the high energy.

4. Cross Sections of 10~100GeV neutrinos

- Deep inelastic scattering
  - Scattering with a quark in a nucleus

  Example: a proton with the ν_μ beam

  - $\nu_\mu + d \rightarrow \mu^- + u$ scattering

$$\frac{d^2\sigma}{dxdy}\sigma(\nu_\mu d \rightarrow \mu^- u) = \frac{G^2 \cos^2\theta_C}{\pi} xs$$

    - x (Feynman x) is the momentum fraction of the quark in the proton.

  - Anti-quarks exist in a proton as a sea quark.

$$\frac{d^2\sigma}{dxdy}\sigma(\nu_\mu \bar{u} \rightarrow \mu^- \bar{d}) = \frac{G^2 \cos^2\theta_C}{\pi} xs(1-y)^2$$

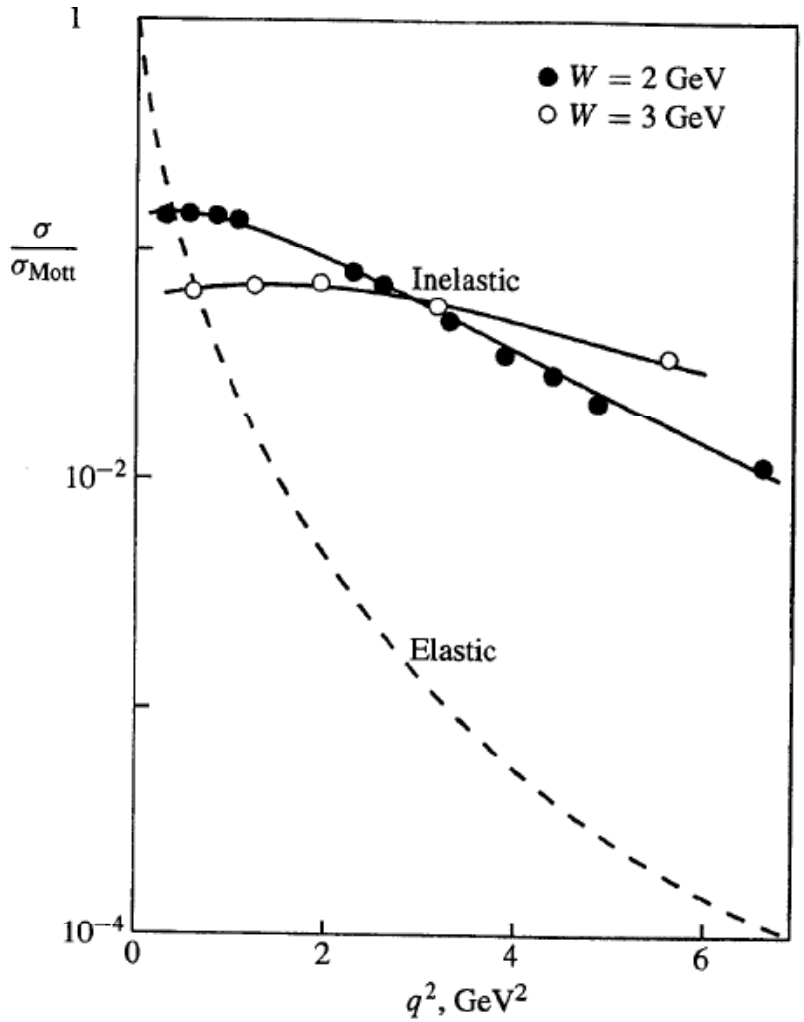


Fig. 5.11. Early SLAC measurements of the inelastic electron–proton scattering cross-section divided by the Mott (pointlike) cross-section, for two values of the invariant mass W of the hadronic final state. The ratio is seen to be only weakly q^2-dependent, in contrast with the strong q^2 dependence of the elastic scattering process, taken from Figure 5.9.

– In the proton, we define the quark distribution function: u(x), d(x), s(x), …. The cross section with a proton is

$$\frac{d^2\sigma}{dxdy}\sigma(\nu_\mu p) = \frac{G^2\cos^2\theta_C}{\pi} xs\left[d(x) + \bar{u}(x)(1-y)^2\right]$$

– In the case of neutron, by using the isospin symmetry ($d^n(x)=u^p(x)$),

$$\frac{d^2\sigma}{dxdy}\sigma(\nu_\mu n) = \frac{G^2\cos^2\theta_C}{\pi} xs\left[u(x) + \bar{d}(x)(1-y)^2\right]$$

– In the case of isoscalar target ($\sum n = \sum p$)

$$\frac{d^2\sigma}{dxdy}\sigma(\nu_\mu N) = \frac{G^2\cos^2\theta_C}{2\pi} xs\left\{\left[u(x) + d(x)\right] + \left[\bar{u}(x) + \bar{d}(x)\right](1-y)^2\right\}$$

– For anti-neutrinos,

$$\frac{d^2\sigma}{dxdy}\sigma(\bar{\nu}_\mu N) = \frac{G^2\cos^2\theta_C}{2\pi} xs\left\{\left[u(x) + d(x)\right](1-y)^2 + \left[\bar{u}(x) + \bar{d}(x)\right]\right\}$$

$$Q = \int x\left[u(x) + d(x)\right]dx\ , \qquad \overline{Q} = \int x\left[\overline{u}(x) + \overline{d}(x)\right]dx\ ,$$

$$\sigma(\nu N) = \frac{G^2 \cos^2\theta_C m_p E}{\pi}\left(Q + \frac{1}{3}\overline{Q}\right)$$

$$\sigma(\overline{\nu} N) = \frac{G^2 \cos^2\theta_C m_p E}{\pi}\left(\overline{Q} + \frac{1}{3}Q\right)$$

$$\sigma(\nu N) \approx 2 \times \sigma(\overline{\nu} N)$$

$$\Rightarrow \overline{Q}/Q \approx 0.15$$

- This example is CC current.

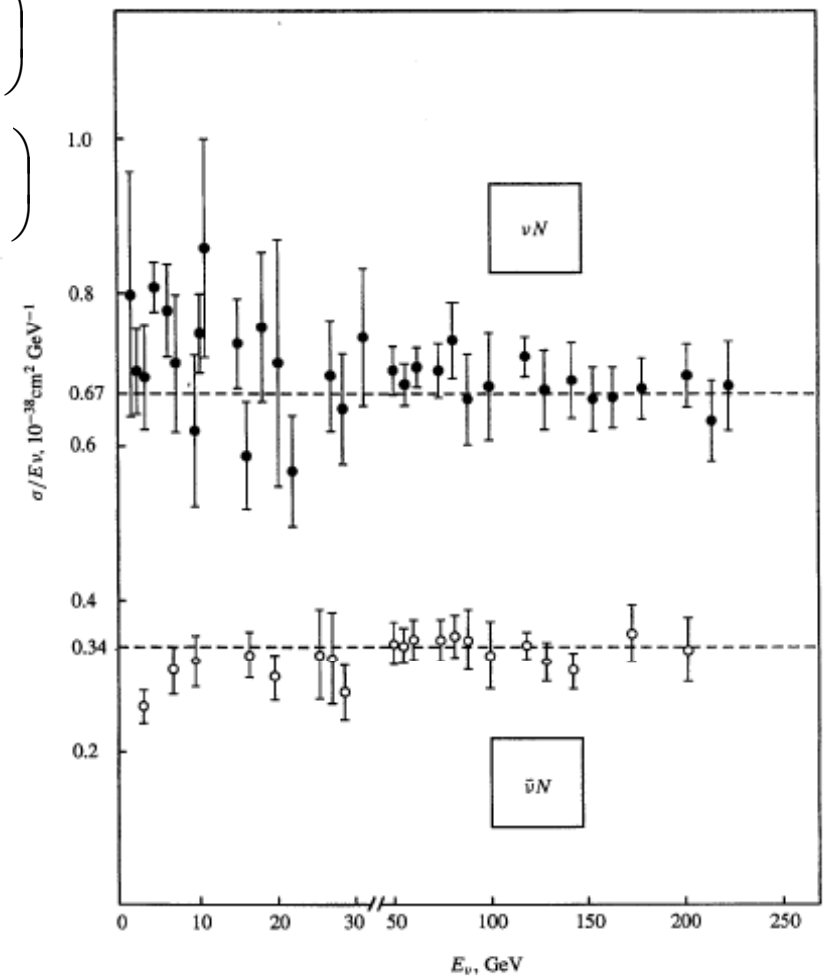


Fig. 5.13. Neutrino and antineutrino cross-sections on nucleons. The ratio σ/E_ν is plotted as a function of energy and is indeed a constant, as predicted in (5.45) and (5.46).

- NC current:

$$\left(\frac{d^2\sigma}{dxdy}\right)_{\nu N \to \nu N} = \frac{G^2 s}{4\pi}\rho\left[xq_1(x) + x(1-y)^2\overline{q}_1(x)\right]$$

$$q_1(x) = 4\left[g_L^2(u) + g_L^2(d)\right]q(x) + 4\left[g_R^2(u) + g_R^2(d)\right]\overline{q}(x)$$

$$\overline{q}_1(x) = 4\left[g_L^2(u) + g_L^2(d)\right]\overline{q}(x) + 4\left[g_R^2(u) + g_R^2(d)\right]q(x)$$

– What is $g_L(u)$, $g_R(u)$, $g_L(d)$, $g_R(d)$?

$$-i\frac{g}{\cos\theta_W}(J_\mu^{\ 3} - \sin^2\theta_W J_\mu^{\ EM})Z^\mu$$

	g_L	g_R
e, μ, τ	$-\frac{1}{2}+\sin^2\theta_W$	$\sin^2\theta_W$
ν	½	0
u,c,t	$\frac{1}{2}-2/3*\sin^2\theta_W$	$-2/3*\sin^2\theta_W$
d,s,b	$-\frac{1}{2}+1/3*\sin^2\theta_W$	$1/3*\sin^2\theta_W$

$$R(\nu N) = \frac{\sigma^{NC}(\nu N \to \nu + X)}{\sigma^{CC}(\nu N \to \mu + X)}$$

$$= \left[1/2 - \sin^2\theta_W + 5/9\,(1 + r)\sin^4\theta_W\right]\rho$$

$$R(\bar{\nu} N) = \frac{\sigma^{NC}(\bar{\nu} N \to \bar{\nu} + X)}{\sigma^{CC}(\bar{\nu} N \to \mu + X)}$$

$$= \left[1/2 - \sin^2\theta_W + 5/9\,(1 + 1/r)\sin^4\theta_W\right]\rho$$

$$, r = \frac{\sigma(\bar{\nu} N \to \mu^+ + X)}{\sigma(\nu N \to \mu^- + X)} = \frac{1/3 + \bar{Q}/Q}{1 + \bar{Q}/3Q}$$

- FNAL NuTeV experiment measured this quantities.

NuTeV results on R^{ν} and $R^{\nu bar}$

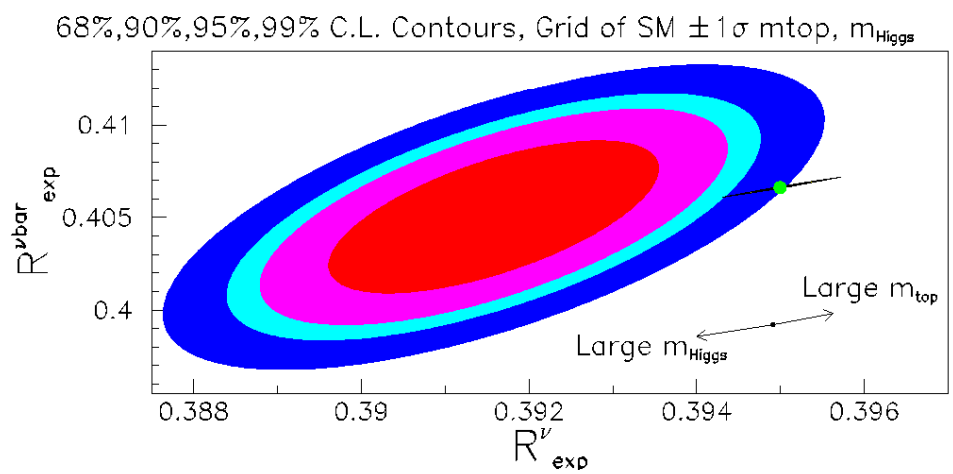
- NuTeV result:

$$\sin^2\theta_W^{(on-shell)} = 0.2277 \pm \pm 0.0013(stat.) \pm 0.0009(syst.)$$
$$= 0.2277 \pm 0.0016$$

- Standard model fit (LEPEWWG): 0.2227 ± 0.00037

A 3σ discrepancy…

$$R^{\nu}_{\exp} = 0.3916 \pm 0.0013$$
$$(SM: 0.3950) \quad \Leftarrow 3\sigma \ difference$$
$$R^{\bar{\nu}}_{\exp} = 0.4050 \pm 0.0027$$
$$(SM: 0.4066) \quad \Leftarrow Good \ agreement$$



5. Cross Sections of >TeV neutrinos

- Neutrinos are an important particle to search for astrophysical sources.
  - Example: ICE-Cube experiment.
  - The energy of neutrino is much higher than that available by an accelerator.
  - Basic interaction is DIS with some corrections.
    - $$G_F^2 \to G_F^2 \frac{1}{\left(1 - q^2 / m_W^2\right)^2} = G_F^2 \frac{1}{\left(1 - 2 m_N E_\nu xy / m_W^2\right)^2}$$
    - Small x (sea quark contribution) is getting important.

$$\sigma_{CC}(\nu N) \approx 2.69 \times 10^{-36}\, cm^2 \left(\frac{E_\nu}{1\,GeV}\right)^{0.402}$$

$$\sigma_{NC}(\nu N) \approx 1.06 \times 10^{-36}\, cm^2 \left(\frac{E_\nu}{1\,GeV}\right)^{0.408}$$

$$\sigma_{CC}(\bar{\nu} N) \approx 2.53 \times 10^{-36}\, cm^2 \left(\frac{E_\nu}{1\,GeV}\right)^{0.404}$$

$$\sigma_{NC}(\bar{\nu} N) \approx 0.98 \times 10^{-36}\, cm^2 \left(\frac{E_\nu}{1\,GeV}\right)^{0.410}$$

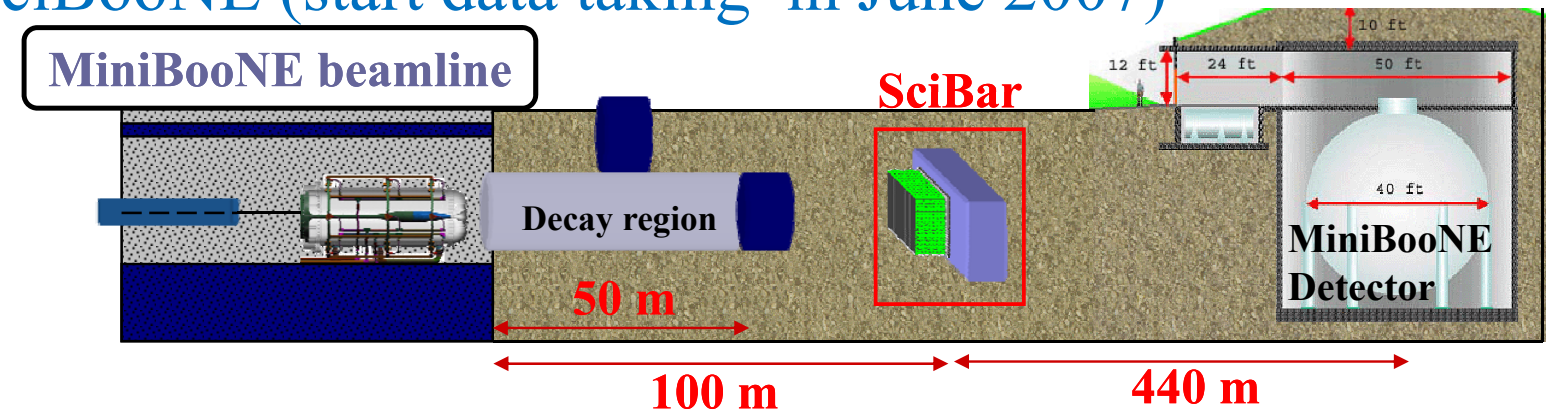
$$\bar{Q}/Q \sim 1$$

HOMEWORK: For an example, how long is the interaction length of 100TeV neutrino in ice?

6. Experiments for neutrino cross sections

- Neutrino detectors in the short baseline oscillation experiment
  - NuTeV, NOMAD, CHORUS, MiniBooNE
- Near neutrino detectors in the long baseline oscillation experiment.
  - K2K, MINOS, T2K, NOνA
- Dedicated experiments to measure the neutrino cross section
  - SciBooNE, MINERνA
  - νSNS
    - study the cross section of ~O(10MeV) for Supernova detection.

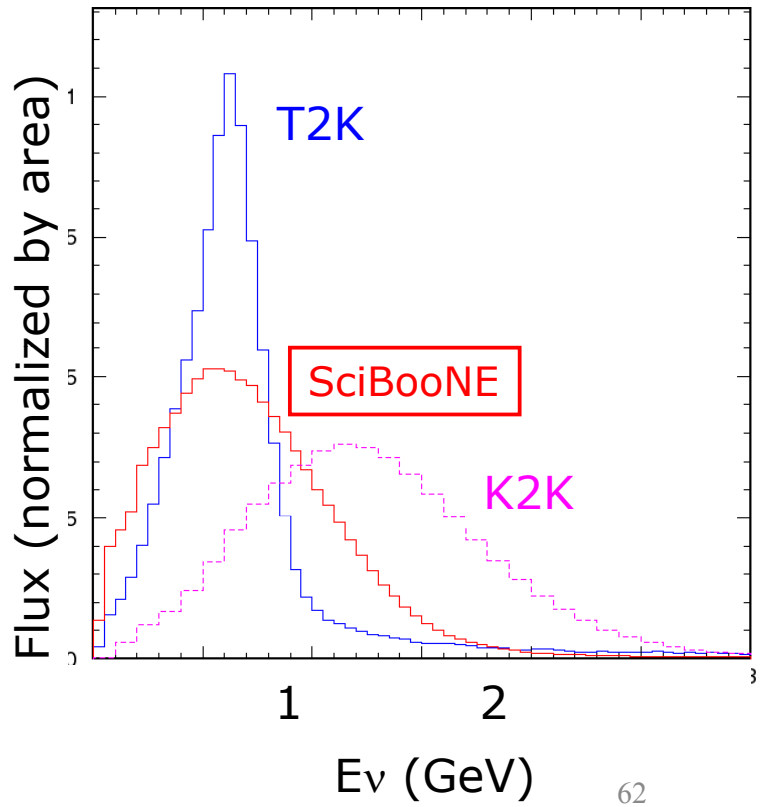
SciBooNE (start data taking in June 2007)



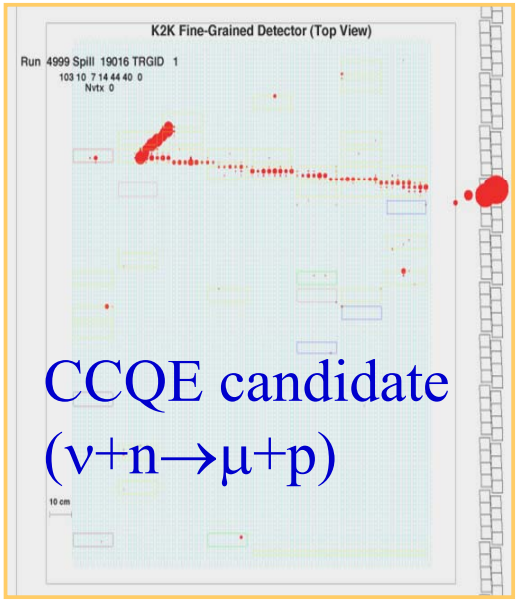
- Precision study of neutrino cross sections for T2K.
- Anti-neutrinos
  - Unexplored physics territory and important for CP study in T2K-II.

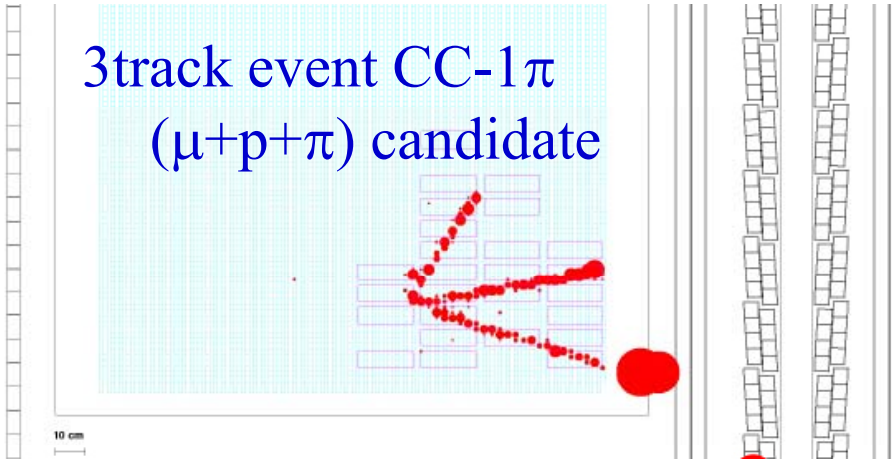
K2K-SciBar + FNAL-BNB

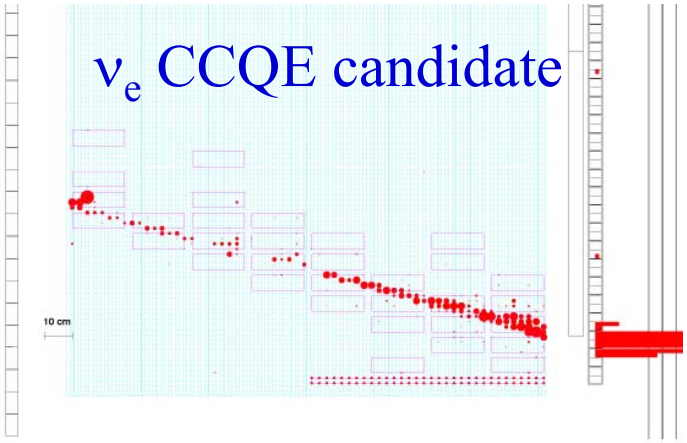
- Well developed Detector
- Most intense low energy neutrinc beam.

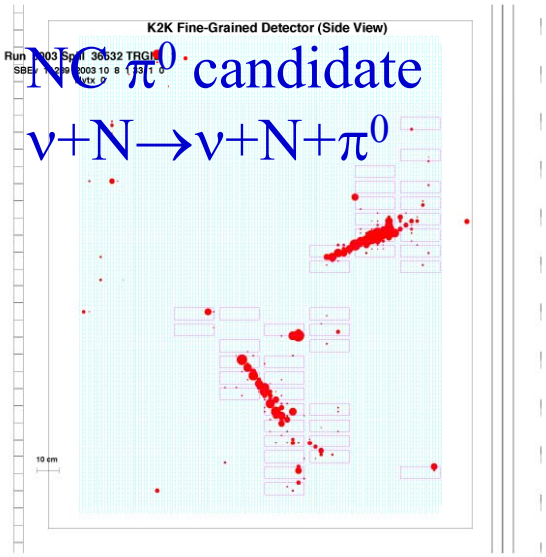


SciBar detector event display in K2K



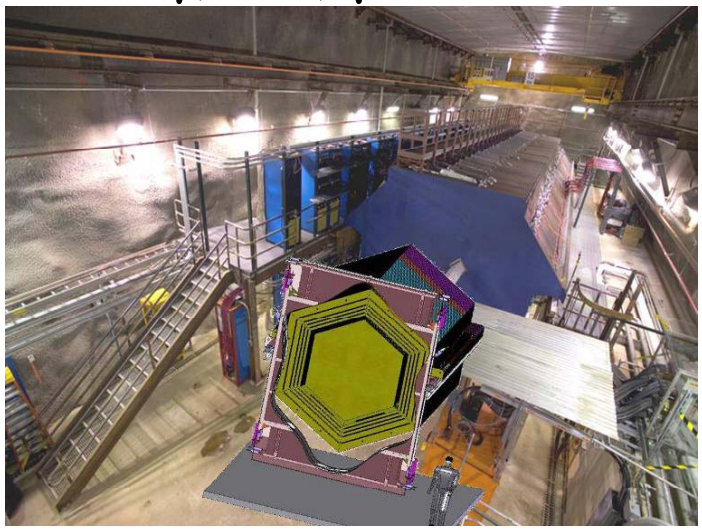


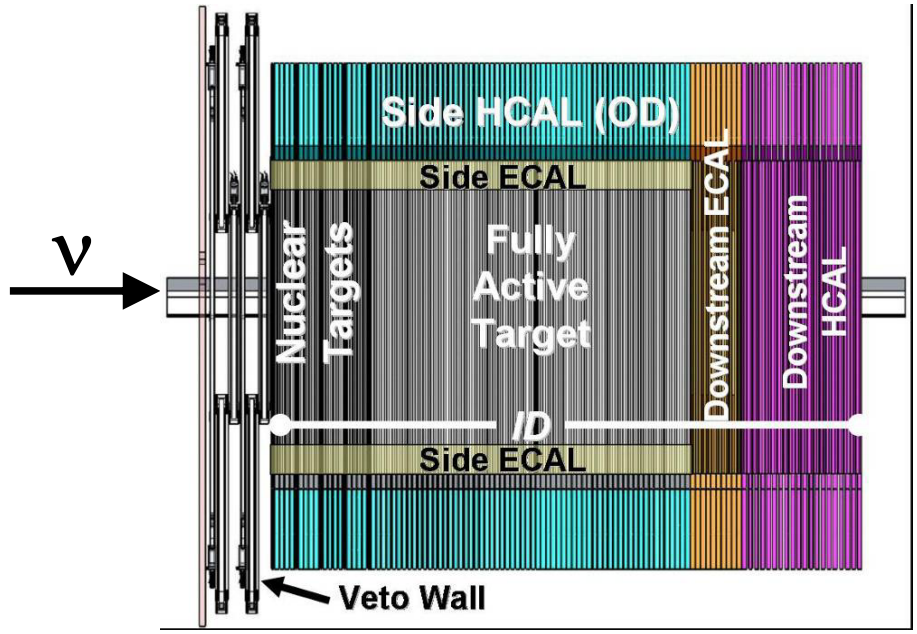




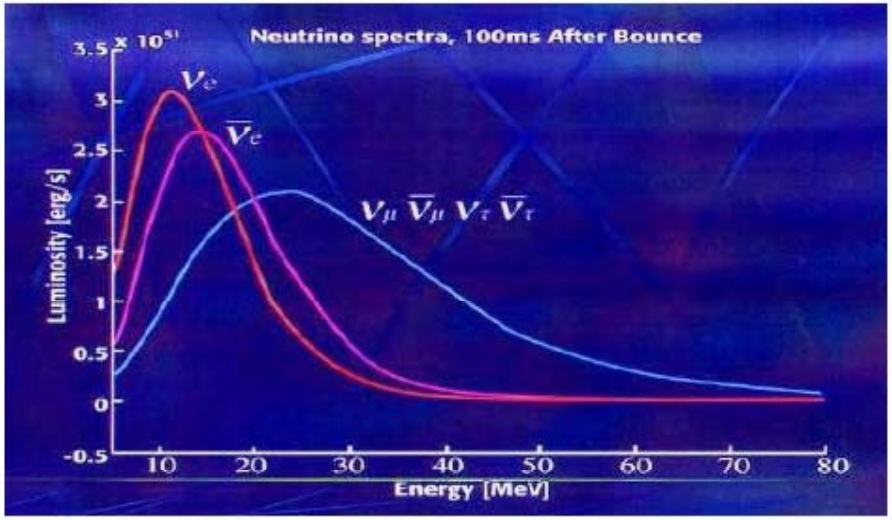
MINERνA at NuMI (start in 2009)

- Need a high granularity detector (like SciBar) but in a higher energy beam and with improved containment of γ, $\pi^{\pm}$, μ





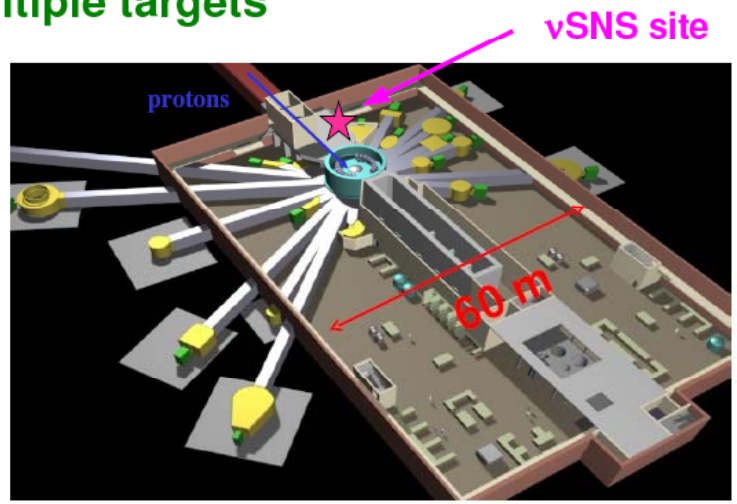
- MINERνA at NuMI
  - "chewy center" (active target)
  - with a crunchy shell of muon, hadron and EM absorbers



Study CC and NC interactions with various nuclei, in few to 10's of MeV range

NuSNS (Neutrinos at the SNS)

A neutrino facility with capability to measure multiple targets



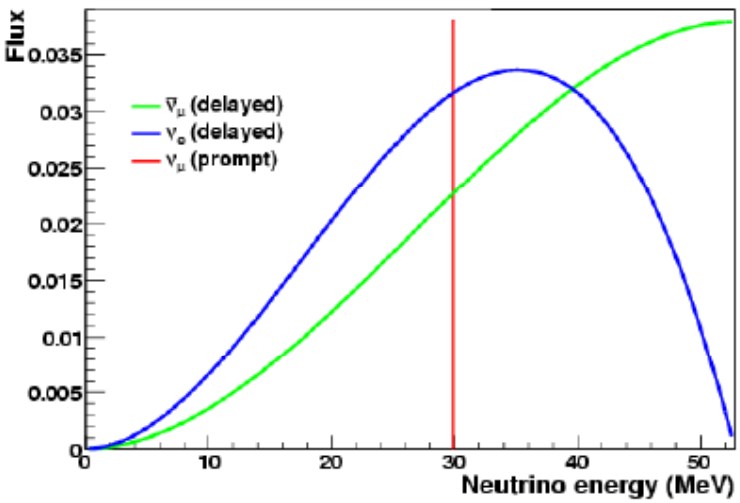
CNS

$$\pi^+ \rightarrow \mu^+ + \nu_\mu$$

2-body decay: monochromatic 29.9 MeV ν_μ
PROMPT

$$\mu^+ \rightarrow e^+ + \bar{\nu}_\mu + \nu_e$$

3-body decay: range of energies between 0 and $m_\mu/2$
DELAYED (2.2 μs)



7. Summary

- Understanding of neutrino cross section is a good exercise to review the standard model.
- In reality, the reliable estimation is not easy because of the nuclear (and nucleon?) structure.
  - Experimental Inputs and model buildings are essential!
- The next generation *precision* oscillation experiments request the precise information of neutrino cross sections.
  - Including the final state kinematics (although I did not fully cover this topics in the lecture).
-

Play together with neutrinos!

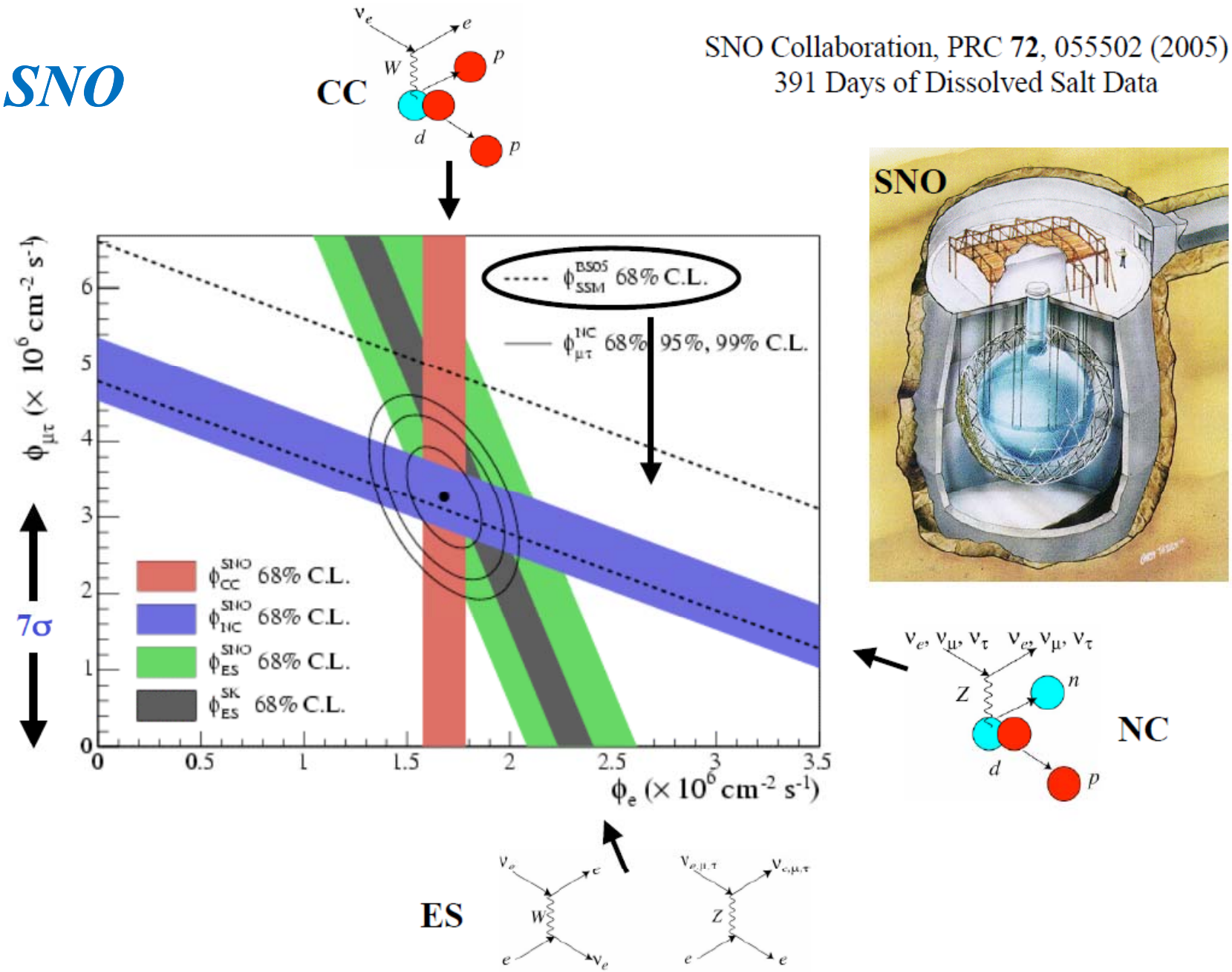
Reference materials

Neutrino Experiments

- KamLAND, Super-K, SNO, Borexino, GNO, Homestake, one more
- LSND, Karmen
- K2K, T2K, Super-K, MINOS, Soudan2, MACRO, MiniBooNE
- NuTeV, CCFR, OPERA, CHORUS, NOMAD
- SciBooNE, MINERnA

SNO

SNO Collaboration, PRC **72**, 055502 (2005)
391 Days of Dissolved Salt Data



1. Introduction

Neutrino Oscillation Experiments.

- Experimental Condition:
  - Solar Neutrinos (ν_e)

    E_ν~ 0.1~15MeV, L~1.5×10^8km, Δm^2 ~>10^{-11}eV^2, σ_ν ~ 10^{-43}cm^2.
  - Reactor Neutrinos ($\bar{\nu}_e$)

    E_ν~ 1~9MeV, L~1~100km, Δm^2 ~>10^{-5}eV^2, σ_ν ~ 10^{-41}cm^2.
  - Atmospheric Neutrinos (ν_e, $\bar{\nu}_e$, ν_μ, $\bar{\nu}_\mu$)

    E_ν~ 20~>10^5MeV, L~10~1×10^4km, Δm^2 ~>10^{-4}eV^2, σ_ν ~ 10^{-42}~10^{-36}cm^2.
  - Accelerator Neutrinos (ν_e, $\bar{\nu}_e$, ν_μ, $\bar{\nu}_\mu$)

    E_ν~ 20~10^5MeV, L~0.01~1000km, Δm^2 ~>10^{-3}eV^2, σ_ν ~ 10^{-42}~10^{-36}cm^2.

- Neutrino Targets:
  - Solar Neutrinos:
    - Gallex(GNO), SAGE, Homesatke, Kamiokande, Super-K, SNO

    Ga, Cl, H_2O, D_2O and e^- ,
  - Reactor Neutrinos:
    - CHOOZ, KamLAND

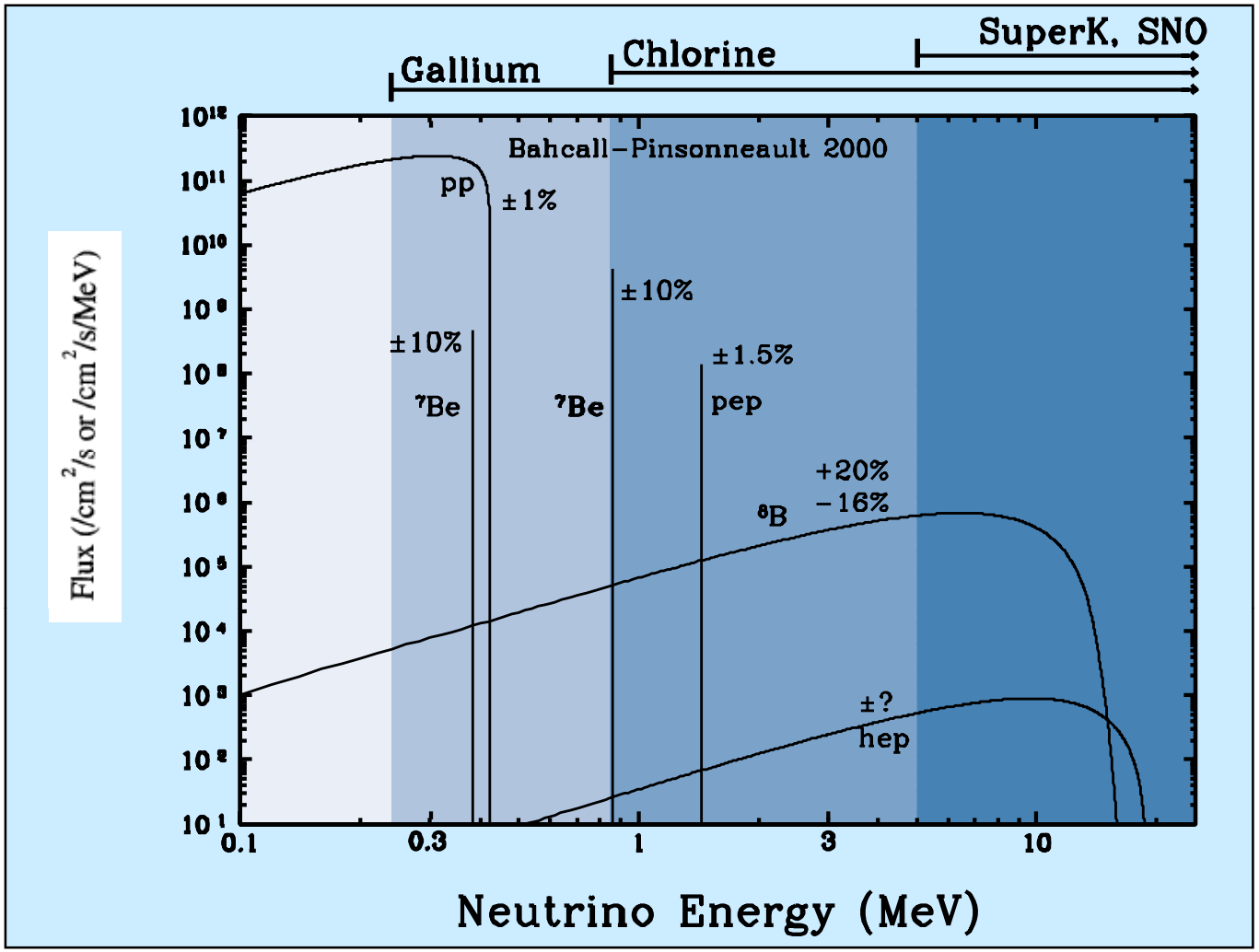
    C (Carbon),
  - Atmospheric Neutrinos:
    - Soudan2, Kamioaknde, Super-K, MINOS, MACRO

    Fe, H_2O , CH
  - Accelerator Neutrinos:
    - LSND, MiniBooNE, K2K, MINOS, OPERA, CCFR, CHORUS, NOMAD

    Carbon, H, Fe, H_2O, and other Nucleus

Neutrino Flux

Solar Neutrino Flux



Plot adapted from http://www.sns.ias.edu/~jnb/

Reactor Neutrinos

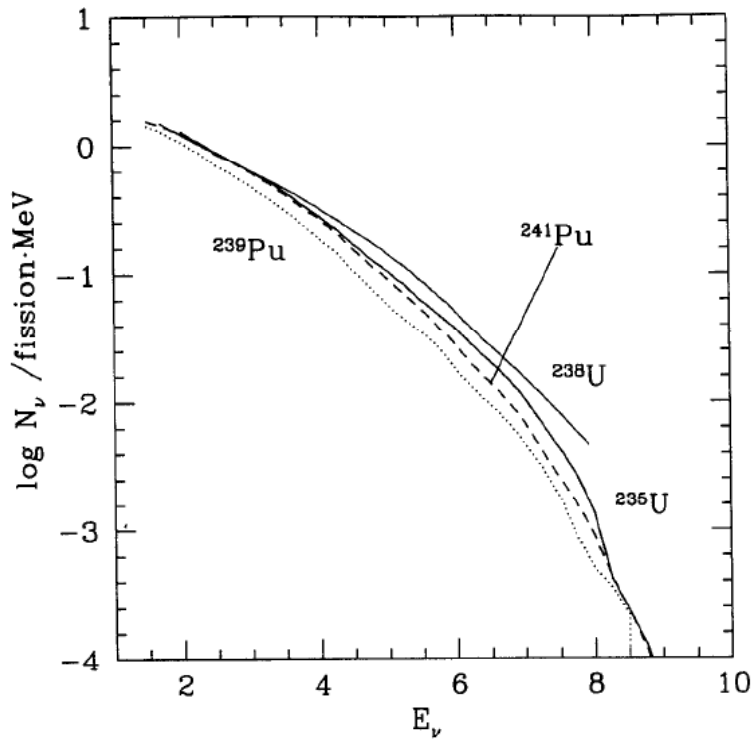


Fig. 4.2. Neutrino spectra from fission of ^{235}U, ^{239}Pu, ^{241}Pu (measured), and ^{238}U (calculated).

- KamLAND Flux: $2\times10^{6}\text{cm}^{-2}\text{s}^{-1}$
  - 70GW reactors at 100 – 250 km away

Accelerator neutrino flux

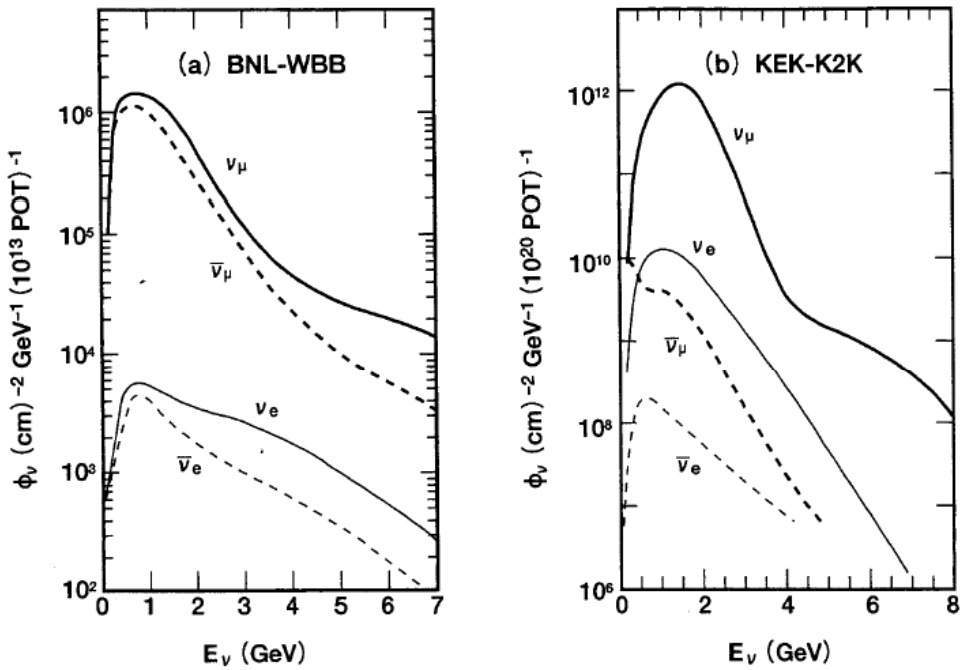


Fig. 4.4. (**a**) Neutrino flux in the Brookhaven AGS wide-band neutrino beam produced from 28.3-GeV protons on a sapphire (Al_2O_3) target with two horns (12 kV×250 kA) which enhance alternatively the neutrino or antineutrino flux. The curve shows the flux at the detector located 110 m away from the second horn with 10^{13} protons incident on target (POT). Data taken from [551]. (**b**) Neutrino flux produced by the KEK 12 GeV PS, used for the K2K long baseline neutrino oscillation experiment at Kamioka. The curve shows the flux at the front detector, which is located at a distance of 300 m from the second horn (operated at 250 kA) which focuses only positively charged particles. 10^{20} POT are the goal number of accumulated protons in the K2K experiment. Data taken from [558].

Atmospheric neutrino flux

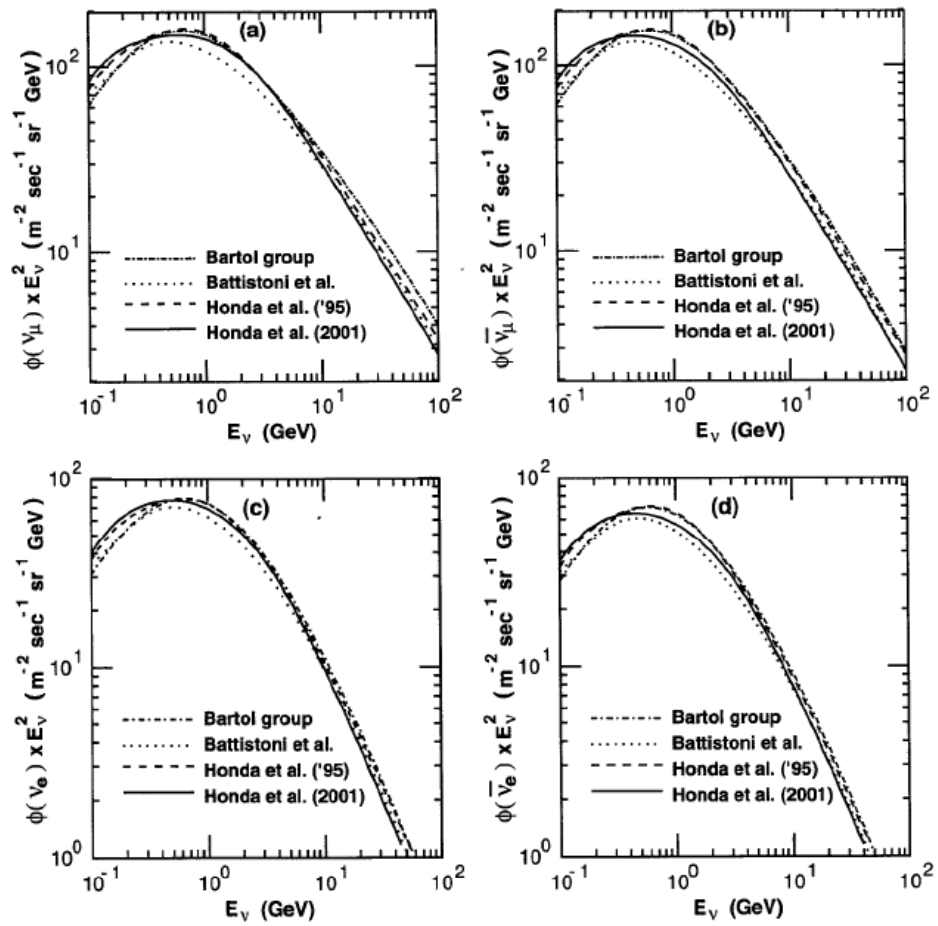


Fig. 4.10. Comparison of the atmospheric neutrino fluxes of BGS [563]/AGLS [564] (Bartol group), Battistoni et al. [572], HKKM [565] (Honda et al. 1995), and Honda et al. 2001 [610]: (**a**) ν_μ flux, (**b**) $\bar{\nu}_\mu$ flux, (**c**) ν_e flux, and (**d**) $\bar{\nu}_e$ flux.

Cosmic Ray flux

Features of atmospheric neutrino flux.

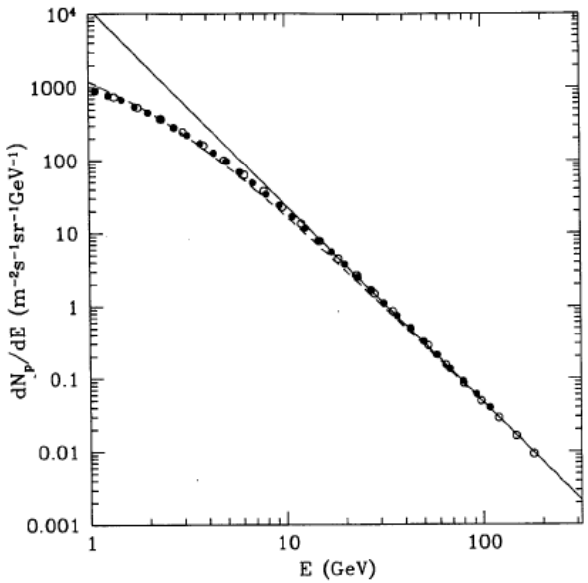


Fig. 4.5. Cosmic-ray proton flux measured by BESS (*solid circles*) and AMS (*open circles*). The *solid curve* represents (4.7). The *dashed curve* that closely matches the data points is the same spectrum but modulated by the solar wind according to the Gleeson–Axford formula (4.9) with $\phi = 750\,\mathrm{MeV}$.

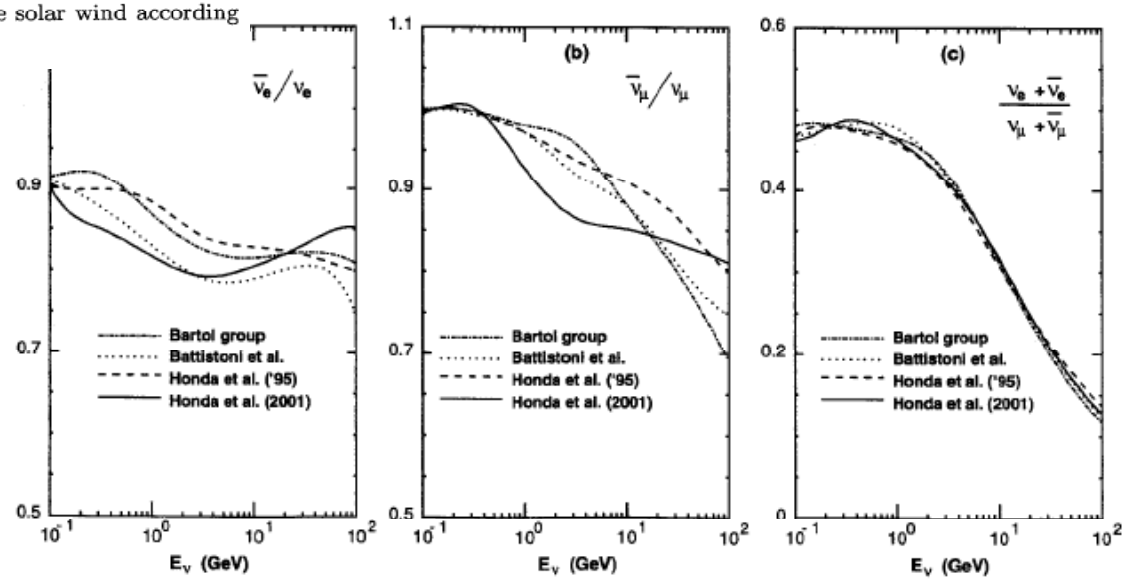


Fig. 4.11. Comparison of the atmospheric neutrino flux calculations for ratios (**a**) $\bar{\nu}_e/\nu_e$, (**b**) $\bar{\nu}_\mu/\nu_\mu$, and (**c**) $(\nu_e+\bar{\nu}_e)/(\nu_\mu+\bar{\nu}_\mu)$. See Fig. 4.10 for references.

Supernova neutrino flux

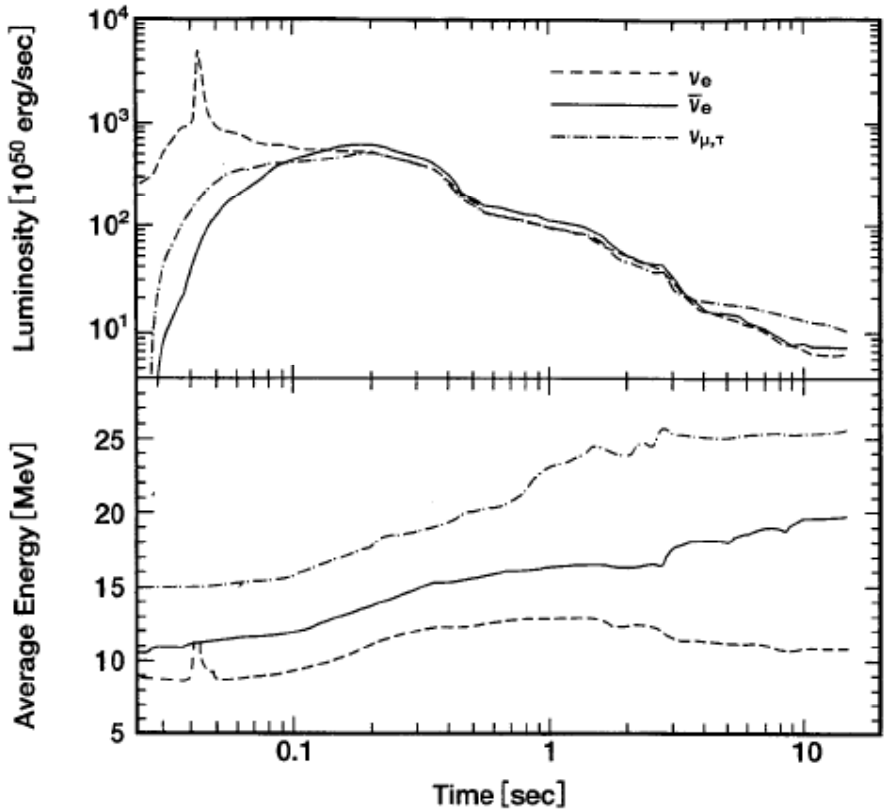


Fig. 4.21. Typical neutrino fluxes form a stellar core collapse, as calculated by Wilson and collaborators. The upper panel shows the luminosity, and lower panel shows the mean energy. $\nu_{\mu,\tau}$ stands for each of ν_μ, ν_τ and their antiparticles. After Totani et al. [784].

Geo-neutrino flux

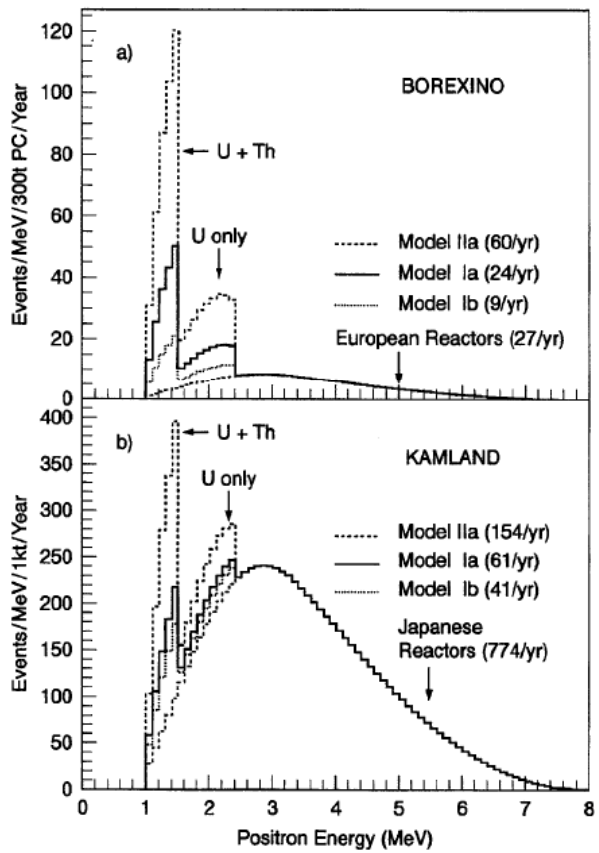


Fig. 4.27. Antineutrino spectrum from ^{238}U and ^{232}Th deposited in Earth's crust, expected in Borexino and KamLAND neutrino detectors. The three curves show different models: Ia and Ib correspond to two different models with the estimated crustal abundance of U/Th [914]; II assumes a full 40 TW ascribed to U/Th radioactivity. Model *a* employs an estimated distribution of crustal U/Th [914], and b assumes a uniform distribution in the crust. After [913].