Dealing with **Nuclear Effects** in Measuring Neutrino Cross Sections and Oscillation Parameters

Neutrino Oscillation Experiments Need Massive Nuclear Targets

**Fermilab/KEK Neutrino Physics Summer School**

Jorge G. Morfín
Fermilab
**Steps (not necessarily time-ordered) in Low Energy $\nu$-Nucleus Interactions**

- Incoming neutrino imparts $q$ (momentum and energy) via an IVB to a nucleus.

- Depending on $q$:
  - the IVB interacts with the entire nucleus, $\geq 1$ nucleon or with one or more quarks.

- Depending on $x_{\text{Bj}}$:
  - The probability of interacting with a quark within the nucleus will be different than in an unbound nucleon (shadowing, anti-shadowing, “EMC” effect).

- For interactions with a nucleon or the quarks within,
  - the target nucleon (off shell) carries initial $p$ and $E_k$ (binding energy).
  - Pauli Blocking influences final state nucleon momentum.

- A proto-hadronic state is created and proceeds through the nucleus before forming a strong interacting hadronic state.
  - Hadronic formation length

- Depending on formation length, the produced hadronic state (quasi-elastic, resonance, continuum, DIS…) proceeds through the nucleus.
  - Nuclear transparency, and nuclear densities influence **final state interactions**.

- A **visible** final hadronic state and **visible** neutrino energy are recorded.
New Concepts introduced by Nuclear Effects

- **Fermi motion:** Since the nucleon is localized to a region of space on the order of 10 fm, it must have some momentum from the uncertainty principle. Typically 100-200 MeV/c.

- **Binding Energy:** In elastic scattering all of the energy transferred from the lepton goes into kinetic energy of the hadron. Now some of it needs to go to removing the nucleon from the nucleus.

- **Fermi Gas Model and Spectral Functions:** Include effects of Fermi motion and binding energy.

- **Pauli Blocking:** Nucleons are fermions and obey Fermi-Dirac statistics which allows only two nucleons per energy level. Scatterings which would take the nucleon to a new state already occupied by other nucleons are not allowed.

- **Hadron Formation Length:** The struck quark (pair) proceeds a distance through the nucleus before forming a strongly interacting particle (pion).

- **Final State Effects:** Any hadrons we produce in the interaction now have to travel through the nucleus before we have any chance of detecting them. Along the way they can interact with other nucleons - intranuclear rescattering.
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Why do we care?
Disappearance experiments

- Predict un-oscillated charged current (CC) spectrum at Far Detector (fixed L)
- Compare with measured *visible energy* spectrum to extract oscillation parameters

\[
P(\nu_\mu \rightarrow \nu_\mu) = 1 - \sin^2 2\theta \sin^2 (1.267 \Delta m^2 L / E)
\]

Incoming Neutrino Energy is NOT Equal to Visible Energy
With Low-Energy Neutrinos, the difference can be significant
Why do we care?
Appearance Experiments

- Backgrounds to $\nu_e$ appearance experiments are, for example, NC $\pi^0$ production where the $\pi^0$ mimics an electron.
- An important input to calculating this background is the cross section for producing this final state.
- However, in a nucleus, final state interactions within nuclear matter will change the number of produced pions of given charge to the number of visible pions.
Nuclear Effects in Neutrino Interactions

- Certain reactions prohibited - Pauli suppression

- Target nucleon in motion - fermi gas and spectral functions

- Hadronically interacting particles are not formed instantaneously

- Produced resonance topologies are modified by final-state interactions reducing detected energy.

- Structure functions are modified and parton distribution functions within a nucleus are different than in an isolated nucleon.
Pauli Exclusion/Suppression Factor

- Two identical fermions cannot coexist in the same energy level within a nucleus.
- Recall that neutrons and protons are treated as non-identical fermions a priori.
- Then, for example (Nakaya’s lecture), each energy level can contain 4 nucleons: 2 protons and 2 neutrons. The protons differ by their z-component of intrinsic spin, and also for the neutrons.
- For the quasi-elastic interaction, that changes a neutron to a proton, enough energy has to be transferred to the proton to avoid this problem or the reaction does not take place.
Pauli Suppression Factor
Nucleon Motion within a Nucleus
Fermi Motion: Fermi Gas and Spectral Functions

Assume target nucleon has no initial momentum in quasi-elastic scattering

\[ \omega = \frac{E^2_{\mu} (1 - \cos \Theta)}{M - E_{\mu} (1 - \cos \Theta)} \]
Target Neutron Momentum in Fe
Spectral Function vs Fermi Gas

Change in reconstructed $Q^2$

Change in reconstructed $E_\nu$
Effect of neutron initial momentum on $\sigma$

Note: Identical cross section model – Net effect of smearing due to fermi motion

- Computed with neutrons having momentum/energy distributions as SpectralFunc1d/C12 (see next)
Hadron Formation Length - hadronization
Will Brooks, Dave Gaskell - Jefferson lab
Models of Hadron Attenuation

- Hadron production from nuclei can be influenced by:
  - Pre–hadronized quark interactions with other nucleons in nucleus
  - Produced hadron interactions with other nucleons

- $t_f = l_f / c$, the hadron formation time will affect which dominates

- One time–scale model - hadron produced “directly”
  \[ \tau_f \approx \frac{E_h R_h}{m_h} \]
  For pion mass, $R_h = 0.66$ fm
  For 0.5 GeV p, $\tau_f \approx 2.4$ fm
We can measure this “formation length”

Experimental Studies: “Hadron Attenuation”

- We can learn about **hadronization distance scales** and **reaction mechanisms** from nuclear DIS
- Nucleus acts as a spatial analyzer for outgoing hadronization products

**HERMES parameterization for pion formation length:**

\[ \tau = 1.4 \cdot \nu \cdot (1 - z) \text{ fm} \]

**Example:** \( z = 0.5, \nu = 9 \text{ GeV} \)

\[ \tau = 6.3 \text{ fm} \sim \text{radius Pb} \]
Pion Formation Length for Lower Energy $\nu$

\[ L_f = \frac{0.342 \ p \ (\text{GeV/c}) \ m_\pi}{m_\pi^2 + a \ p_t^2} \]

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>A</th>
<th>$r_0$ (=1.2 fm $A^{1/3}$)</th>
<th>$p$ ($l_f &gt; r_0$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>12</td>
<td>2.7 (fm)</td>
<td>1.1 (GeV/c)</td>
</tr>
<tr>
<td>O</td>
<td>16</td>
<td>3.0</td>
<td>1.2</td>
</tr>
<tr>
<td>Fe</td>
<td>56</td>
<td>4.6</td>
<td>1.8</td>
</tr>
<tr>
<td>Pb</td>
<td>207</td>
<td>7.1</td>
<td>2.9</td>
</tr>
</tbody>
</table>
Final State Interactions

- Two step process

1. **single pion production** in $\nu N$ scattering
   - Pauli Principle, Fermi motion
2. **multiple scattering** of pions
   - Charge exchange, absorption. Pauli Principle

- step 2 is described by the **charge exchange matrix** $M$
  - only depends on properties of the target
    - charge density profile $\rho(r)$
Charge exchange interaction of initial pion
Paschos

\[ \pi N \text{ reactions } \rightarrow \text{charge exchange:} \]

\[ \pi^+ + N \rightarrow \pi^+ + N, \quad \pi^+ + n \rightarrow \pi^0 + p \]
\[ \pi^0 + N \rightarrow \pi^0 + N, \quad \pi^0 + p \rightarrow \pi^+ + n \]
\[ \pi^0 + n \rightarrow \pi^- + p, \quad \pi^- + N \rightarrow \pi^- + N \]
\[ \pi^- + p \rightarrow \pi^0 + n, \]
\[ \pi^{\pm,0} + N \rightarrow X, \quad (\pi \not\in X) (\rightarrow \text{absorption}) \]

In neutrino scattering

- \( \pi^+ \) cross section is largely reduced (up to 40%) ↔ charge exchange \( M \)
- \( \pi^0 \) cross sections is slightly increased by the nuclear corrections
Parton Distribution Functions within a Nucleus are Different than within a Nucleon

- $F_2$ / nucleon changes as a function of $A$. Measured in $\mu/e - A$, not in $\nu - A$.

- Good reason to consider nuclear effects are DIFFERENT in $\nu - A$.

- Presence of axial-vector current.

- Different nuclear effects for valence and sea → different shadowing for $x_F$ compared to $F_2$.
Summary

- Neutrino oscillation experiments need heavy nuclear targets to collect sufficient statistics for determining oscillation parameters.

- Working in the nuclear environment is “messy” but quantifiable.

- Cross sections and other observables (such as $E_{\text{vis}}$) measured on nucleon targets will be modified in the nuclear environment and the consequences have to be carefully taken into account.

- We need, and will have, experiments that carefully look at the effects of a nuclear environment on neutrino interactions.